

Section 2.3 :

$$\begin{aligned}
 22. \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1} = \lim_{h \rightarrow 0} \frac{(1+h) - 1}{h(\sqrt{1+h} + 1)} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{1+h} + 1)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{\sqrt{1+1}} = \frac{1}{2}
 \end{aligned}$$

$$26. \lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right) = \lim_{t \rightarrow 0} \frac{(t^2 + t) - t}{t(t^2 + t)} = \lim_{t \rightarrow 0} \frac{t^2}{t \cdot t(t + 1)} = \lim_{t \rightarrow 0} \frac{1}{t + 1} = \frac{1}{0 + 1} = 1$$

36. We have $\lim_{x \rightarrow 1} (2x) = 2(1) = 2$ and $\lim_{x \rightarrow 1} (x^4 - x^2 + 2) = 1^4 - 1^2 + 2 = 2$. Since $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x ,

$\lim_{x \rightarrow 1} g(x) = 2$ by the Squeeze Theorem.

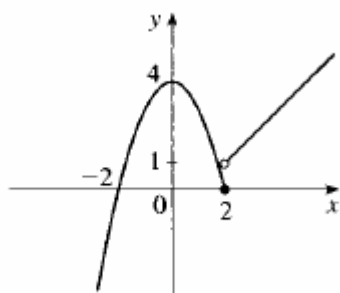
42. Since $|x| = -x$ for $x < 0$, we have $\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x} = \lim_{x \rightarrow -2} \frac{2 - (-x)}{2 + x} = \lim_{x \rightarrow -2} \frac{2 + x}{2 + x} = \lim_{x \rightarrow -2} 1 = 1$.

$$46. (a) \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (4 - x^2) = \lim_{x \rightarrow 2^-} 4 - \lim_{x \rightarrow 2^-} x^2 = 4 - 4 = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x - 1) = \lim_{x \rightarrow 2^+} x - \lim_{x \rightarrow 2^+} 1 = 2 - 1 = 1$$

(b) No, $\lim_{x \rightarrow 2} f(x)$ does not exist since $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$.

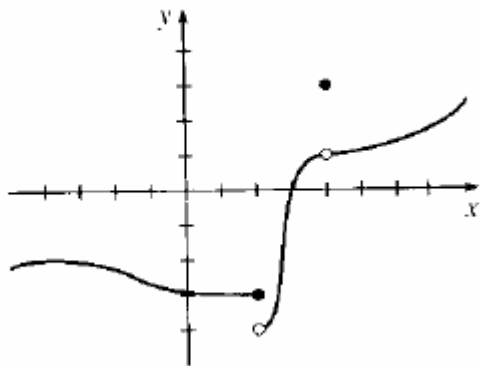
(c)



Section 2.5:

4. g is continuous on $[-4, -2)$, $(-2, 2)$, $[2, 4)$, $(4, 6)$, and $(6, 8)$.

6.



8. (a) Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.
- (b) Continuous; the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.
- (c) Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values—at a cliff, for example.
- (d) Discontinuous; as the distance traveled increases, the cost of the ride jumps in small increments.
- (e) Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

$$12. \lim_{t \rightarrow 1} h(t) = \lim_{t \rightarrow 1} \frac{2t - 3t^2}{1 + t^3} = \frac{\lim_{t \rightarrow 1} (2t - 3t^2)}{\lim_{t \rightarrow 1} (1 + t^3)} = \frac{2 \lim_{t \rightarrow 1} t - 3 \lim_{t \rightarrow 1} t^2}{\lim_{t \rightarrow 1} 1 + \lim_{t \rightarrow 1} t^3} = \frac{2(1) - 3(1)^2}{1 + (1)^3} = \frac{-1}{2} = h(1).$$

By the definition of continuity, h is continuous at $a = 1$.

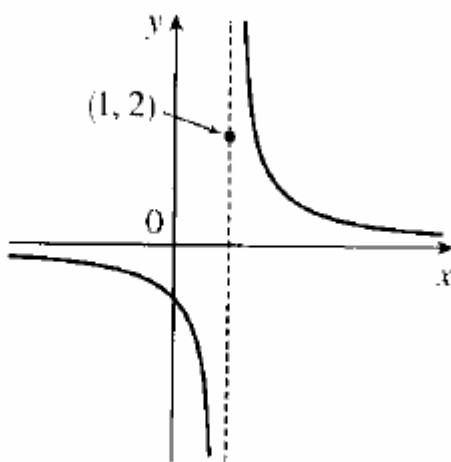
14. For $a < 3$, we have $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} 2\sqrt{3-x} = 2 \lim_{x \rightarrow a} \sqrt{3-x}$ [Limit Law 3] $= 2\sqrt{\lim_{x \rightarrow a} (3-x)}$ [11]

$$= 2\sqrt{\lim_{x \rightarrow a} 3 - \lim_{x \rightarrow a} x}$$
 [2] $= 2\sqrt{3-a}$ [7 and 8] $= g(a)$, so g is continuous at $x = a$ for every a in $(-\infty, 3)$.

Also, $\lim_{x \rightarrow 3^-} g(x) = 0 = g(3)$, so g is continuous from the left at 3. Thus, g is continuous on $(-\infty, 3]$.

16. $f(x) = \begin{cases} 1/(x-1) & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ is discontinuous at 1 because $\lim_{x \rightarrow 1} f(x)$

does not exist.



24. By Theorem 7, the trigonometric function $\sin x$ and the polynomial function $x + 1$ are continuous on \mathbb{R} .

By part 5 of Theorem 4, $h(x) = \frac{\sin x}{x+1}$ is continuous on its domain, $\{x \mid x \neq -1\}$.

32. Because x is continuous on \mathbb{R} , $\sin x$ is continuous on \mathbb{R} , and $x + \sin x$ is continuous on \mathbb{R} , the composite function

$$f(x) = \sin(x + \sin x) \text{ is continuous on } \mathbb{R}, \text{ so } \lim_{x \rightarrow \pi} f(x) = f(\pi) = \sin(\pi + \sin \pi) = \sin \pi = 0.$$

$$36. f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$$

By Theorem 7, the trigonometric functions are continuous. Since $f(x) = \sin x$ on $(-\infty, \pi/4)$ and $f(x) = \cos x$ on

$(\pi/4, \infty)$, f is continuous on $(-\infty, \pi/4) \cup (\pi/4, \infty)$. $\lim_{x \rightarrow (\pi/4)^-} f(x) = \lim_{x \rightarrow (\pi/4)^-} \sin x = \sin \frac{\pi}{4} = 1/\sqrt{2}$ since the sine

function is continuous at $\pi/4$. Similarly, $\lim_{x \rightarrow (\pi/4)^+} f(x) = \lim_{x \rightarrow (\pi/4)^+} \cos x = 1/\sqrt{2}$ by continuity of the cosine function at

$\pi/4$. Thus, $\lim_{x \rightarrow (\pi/4)} f(x)$ exists and equals $1/\sqrt{2}$, which agrees with the value $f(\pi/4)$. Therefore, f is continuous at $\pi/4$, so

f is continuous on $(-\infty, \infty)$.

$$38. f(x) = \begin{cases} x + 1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$$

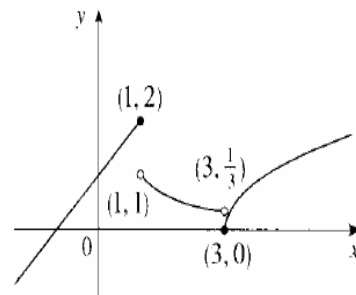
f is continuous on $(-\infty, 1)$, $(1, 3)$, and $(3, \infty)$, where it is a polynomial,

a rational function, and a composite of a root function with a polynomial,

respectively. Now $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x + 1) = 2$ and $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (1/x) = 1$, so f is discontinuous at 1.

Since $f(1) = 2$, f is continuous from the left at 1. Also, $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3^-} (1/x) = 1/3$, and

$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \sqrt{x-3} = 0 = f(3)$, so f is discontinuous at 3, but it is continuous from the right at 3.



48. $f(x) = \sqrt[3]{x} + x - 1$ is continuous on the interval $[0, 1]$, $f(0) = -1$, and $f(1) = 1$. Since $-1 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sqrt[3]{x} + x - 1 = 0$, or $\sqrt[3]{x} = 1 - x$, in the interval $(0, 1)$.