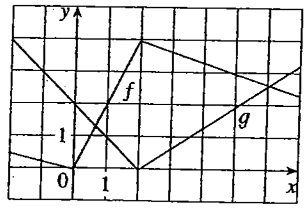
1(a) (5 points) If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.

(i) Find $u'(1)$.

(ii) Find $v'(5)$.



$$u'(1) = f'(1)g(1) + f(1)g'(1)$$
$$= 2 \cdot 1 + 2 \cdot (-1)$$
$$= \boxed{0}.$$

$$v'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{(g(5))^2}$$
$$= \frac{2 \cdot (-\frac{1}{3}) - 3 \cdot (\frac{2}{3})}{4} = \boxed{-2/3}.$$

(b) (5 points) Some values of two functions F and G are given in the table below.

x	F(x)	G(x)	F'(x)	G'(x)
1	2	-1	4	2
2	8	1	0	3
3	2	5	3	1
4	5	6	1/2	-1
5	-3	8	-1/2	-4

Calculate each of the following:

(i) $h'(3)$ where $h(x) = F(x)G(x)$

(ii) $k'(1)$ where $k(x) = F(x)/G(x)$

$$h'(3) = F'(3)G(3) + F(3)G'(3)$$
$$= 3 \cdot 5 + 2 \cdot 1 = \boxed{17}$$

$$k'(1) = \frac{G(1)F'(1) - F(1)G'(1)}{(G(1))^2}$$

$$= \frac{(-1) \cdot 4 - 2 \cdot 2}{1} = \boxed{-8}$$

2. (10 points) Find $f'(x)$ for each function $f(x)$ below. Show details of your work.

(a) $f(x) = 5\sin(x)(\cos(x) + x)$

$$f'(x) = 5\sin(x)(-\sin(x) + 1)$$
$$+ 5\cos(x)(\cos(x) + x)$$

(b) $f(x) = \tan(x) - \frac{4}{x^2}$ $= \tan(x) - 4x^{-2}$

$$f'(x) = \sec^2(x) + 8x^{-3}$$

(c) $f(x) = \sin(x - x^{-1})$

$$f'(x) = \cos(x - x^{-1})(1 + x^{-2})$$

3. (10 points) A certain type of crystal grows in the shape of a cube.

(a) If V is the volume of the cube with side length x, find $\frac{dV}{dx}$ when $x = 3$.

$V(x) = x^3$

$\frac{dV}{dx} = 3x^2$

when $x=3$, $\frac{dV}{dx}$ is $3(3)^2 = \boxed{27 \text{ units}^3}$

(b) What is the surface area of the cube? How does this compare with the rate of change of the volume of the cube with respect to the edge length?

Surface Area $= 6x^2$

rate of change of volume $\left(= \frac{dV}{dx}\right)$ is one half the surface area.

4. (10 points) Let $f(x) = x\sqrt{5-x}$. $= x(5-x)^{1/2}$

(a) Find the equation of the tangent line to the curve $y = f(x)$ at the point $(4, 4)$.

$$f'(x) = x\left(\tfrac{1}{2}(5-x)^{-1/2}(-1)\right) + (5-x)^{1/2}$$

$$= \frac{-x}{2\sqrt{5-x}} + \sqrt{5-x}.$$

$$f'(4) = \frac{-4}{2\sqrt{1}} + \sqrt{1} = -2+1 = -1.$$

So $y-4 = -(x-4)$, or

$$\boxed{y = -x+8}$$

(b) For which values of x is the tangent line horizontal?

This occurs when $f'(x) = 0$:

$$0 = \frac{-x}{2\sqrt{5-x}} + \sqrt{5-x}$$

$$\frac{x}{2\sqrt{5-x}} = \sqrt{5-x}$$

$$x = 2(5-x) = 10-2x$$

$$3x = 10$$

$$\boxed{x = 10/3}$$

5(a) (5 points) Find the following limit by recognizing it as the derivative of a function:

$$\lim_{h\to 0} \frac{\cot(\frac{\pi}{4}+h)-1}{h}$$

Since $\cot(\pi/4)=1$, this limit is the derivative of $\cot(x)$ at $x=4$.

So: limit above $= -\csc^2(\pi/4)$

$$= \frac{-1}{\sin^2(\pi/4)} = \frac{-1}{(\frac{\sqrt{2}}{2})^2}$$

$$= \boxed{-2}$$

(b) (5 points) Find the following limit:

$$\lim_{t\to 0} \frac{\sin(3t)}{-5t}$$

$$\frac{\sin(3t)}{-5t} = \frac{\sin(3t)}{3t} \cdot \frac{3}{-5} \quad \text{so}$$

$$\lim_{t\to 0} \frac{\sin(3t)}{-5t} = \frac{-3}{5} \lim_{t\to 0} \frac{\sin(3t)}{3t} = \frac{-3}{5} \lim_{3t\to 0} \frac{\sin(3t)}{3t}$$

$$= \boxed{\frac{-3}{5}}.$$

6. (10 points) The graph of a function $f(x)$ is given below. Carefully draw the graph of $f'(x)$. Add numbers to the vertical scale of your graph so that the numerical values of $f'(x)$ are reasonably accurate.