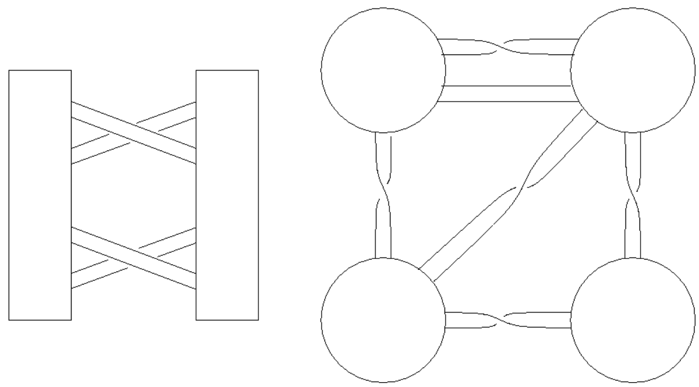
Math 5863 homework

14. (2/8) Use handle slides to simplify and identify each of these two surfaces:



15. (2/8) A space X is defined to be *contractible* if the identity map of X is homotopic to a constant map. Prove the following:

    1. If C is a convex subset of $\mathbb{R}^n$ (in particular, C could be $\mathbb{R}^n$ or D^n), then C is contractible.
    2. X is contractible if and only if there is a retraction from $C(X)$ to X.
    3. If X is contractible, then any map from X to any space Y is homotopic to a constant map.
    4. If X is contractible, then any map from any space Y to X is homotopic to a constant map.
    5. If X is contractible, then two maps $f, g\colon X \to Y$ are homotopic if and only if their images lie in the same path component of Y.

16. (2/8) Use the Classification Theorem to prove that a (compact, connected) surface is planar if and only if it is a connected sum of disks. Hint: First show that if F has a torus or projective plane summand, then it contains a pair of imbedded circles that intersect at only one point, at which they cross. Use the Jordan Curve Theorem to show that $\mathbb{R}^2$ contains no such pair. Use these two facts, plus the Classification Theorem, to show that the only surfaces that imbed in $\mathbb{R}^2$ are connected sums of disks.