Math 5863 homework

8. (2/1) Suppose that $h_0, h_1 \colon X \to Y$ are isotopic. Prove that if $g \colon Y \to Z$ is a homeomorphism, then $g \circ h_0$ is isotopic to $g \circ h_1$. Prove that if $k \colon Z \to X$ is a homeomorphism, then $h_0 \circ k$ is isotopic to $h_1 \circ k$.

9. (2/1) An imbedding $j \colon I \to I$ is called *order-preserving* if $j(0) < j(1)$, otherwise it is called *order-reversing.*

   1. Prove that if j is order-preserving, then $j(x_1) < j(x_2)$ whenever $x_1 < x_2$.
   2. Prove that there are exactly two isotopy classes of imbeddings of I into I, by showing that $j_0, j_1 \colon I \to I$ are isotopic if and only if they are both order-preserving or both order-reversing.

10. (2/1) Prove the Disk Lemma for $n = 1$ and $M = I$. That is, prove that if $j_1, j_2 \colon I \to I$ are imbeddings with image in the interior of I, then j_1 is ambiently isotopic to either j_2 or $j_2 \circ \rho$. Hint: this follows quickly from the fact that any two homeomorphisms of I are isotopic. Compose j_1 and/or j_2 by ρ to assume that both are order preserving. Extend the homeomorphism $j_2 \circ j_1^{-1} \colon j_1(I) \to j_2(I)$ to an order-preserving homeomorphism $h \colon I \to I$, by using linear maps on $\overline{I - j_1(I)}$. Now make use of the fact that id_I and h are isotopic.

11. (2/1) A compact (connected) surface F is called *planar* if $F \neq S^2$ and F can be imbedded into S^2. Show that if F_1 and F_2 are planar, then the connected sum $F_1 \# F_2$ is planar. Hint: Let $D_1 \subset F_1$ and $D_2 \subset F_2$ be admissible disks. Use the Disk Lemma to show that there is an imbedding of F_1 in S^2 that carries D_1 to the upper hemisphere, and there is an imbedding of F_2 in S^2 that carries D_2 to the lower hemisphere.

12. (2/8) Let F and G be compact connected 2-manifolds with nonempty boundary. Let α be an arc (an imbedded copy of I) in ∂F, and β an arc in ∂G. Define the *boundary-connected sum* of F and G to be the surface $F \natural G$ obtained from $F \cup G$ by identifying α and β by a homeomorphism. Draw pictures to explain why $(F \# D^2) \natural (G \# D^2) = F \# G \# D^2$.

13. (2/8) Use the previous problem to give a quick explanation of why attaching a twisted 1-handle to a boundary circle of M produces $M \# P$, and why attaching two untwisted 1-handles with alternating ends to a boundary circle of M produces $M \# T$.