Mathematics 1823-010
Examination III Form B
April 20, 2001

Name (please print) ____________________

Student Number ________________

Discussion Section (please circle day and time): (1)

We 2:30 We 3:30 Th 9:00 Th 10:30 Th 12:00 Th 1:30

I. (7) Let $f(x) = \dfrac{x}{x+2}$ on the domain $1 \le x \le 4$ (i. e. the closed interval $[1, 4]$). Find all values of c that satisfy the conclusion of the Mean Value Theorem for this function on this interval.

II. (7) For the function $f(x) = \dfrac{(1-x^2)(1-x)}{x^3}$, carry out the following.

1. Determine all vertical asymptotes of the graph of $f(x)$.

2. Determine all horizontal asymptotes of the graph of $f(x)$.

3. Determine all values of x at which $f(x)$ changes sign.

III. (10) Calculate the following limits. (To obtain credit, you must show your reasoning, not just guess the limit by computing values on a calculator.)

1. $\displaystyle\lim_{x\to-\infty} \frac{(1-x)(2+x)}{(1+2x)(2-3x)}$

2. $\displaystyle\lim_{x\to\infty} \sqrt{x^2+x} - x$

IV. (4) State the Mean Value Theorem.

V. (4) Let $f(x)$ be a function and let a be an x-value in the domain of $f(x)$. Define what it means to say that f has a *local maximum* at $x = a$, and define what it means to say that f has an *absolute maximum* at $x = a$.

VI. A certain function $f(x)$ has $f'(x) = 2x^2 - 6$. Carry out the following.
(6)

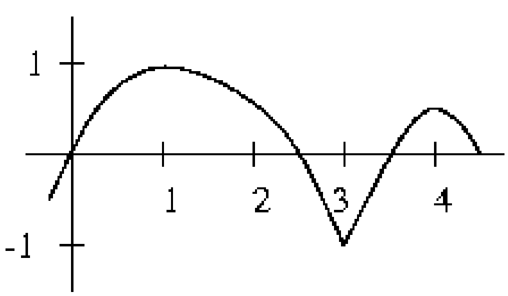
1. Find all critical points of f.

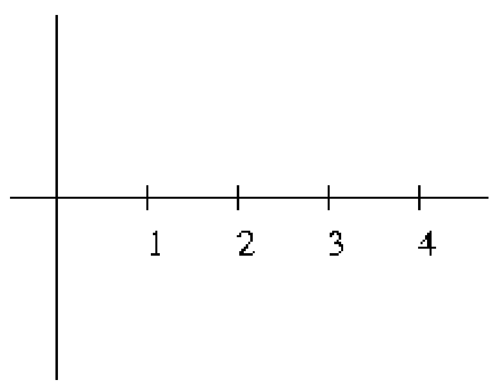
2. By examining $f'(x)$, find all local maxima of f.

3. By examining $f'(x)$, find all local minima of f.

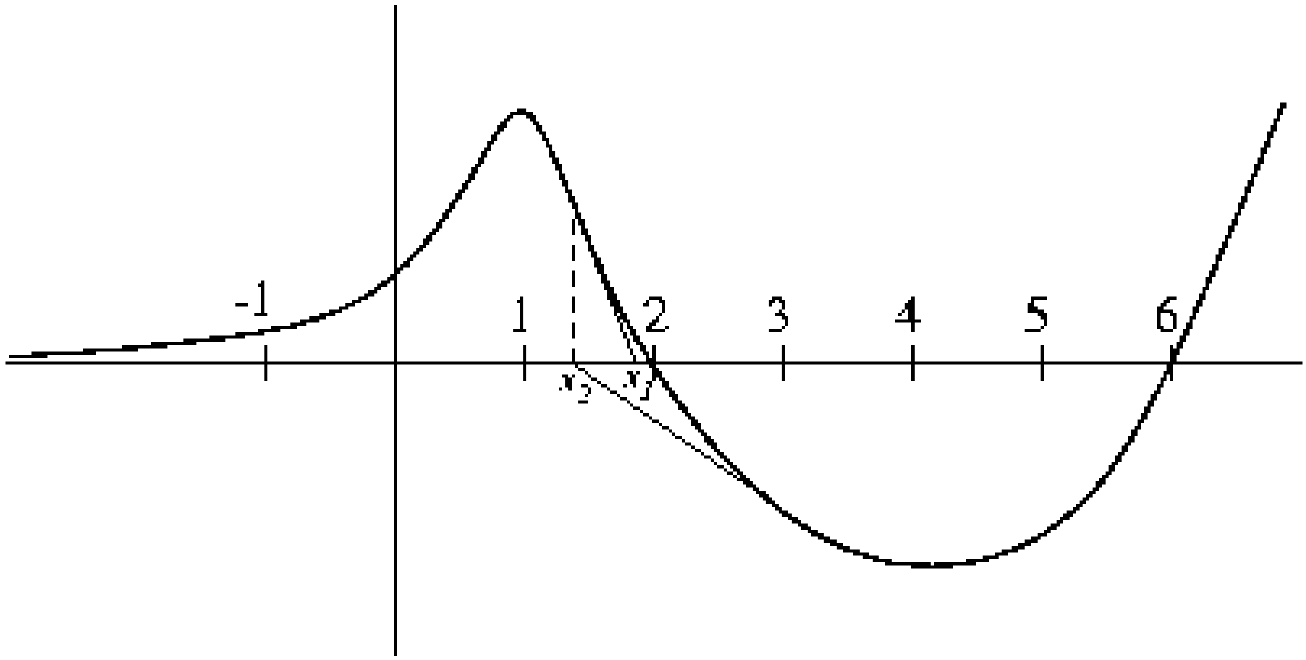
VII. A certain function $g(x)$ has $g''(x) = \dfrac{x}{x+5}$. By examining $g''(x)$, find where g is concave up, and where it is concave down.
(3)

VIII. The first coordinate system below shows the graph of a function $f(x)$. On the second coordinate system, sketch the graph of its derivative $f'(x)$.
(5)





IX. (6) The coordinate system shown here is the graph of a certain differentiable function which has roots at $x = 2$ and $x = 6$, and has a local maximum at $x = 1$ and a local minimum at $x = 4$. For each of the choices of the value of x_1 given below, draw (if possible) the appropriate tangent lines and the points x_2 and x_3 that result when Newton's method is carried out using the starting value x_1. For each choice of x_1, tell what the iteration would do if continued. The problem is already worked out for $x_1 = 3$, as an example.



Example: $x_1 = 3$

The iteration would converge to the root $x = 2$.

1. $x_1 = 0$

2. $x_1 = 1$

3. $x_1 = 5$