Math 1823 homework

Instructions: Work the assigned problems. Book problems shown in **boldface** should be written up formally and turned in no later than the due date.

1. (due 8/31) Section 1.3 # 1, 2, 3, 6, 7, 9, 10, 11-16, 17-21, 22-24, 28, 37-40, 45-50, 52-55, 57-59, 61-64

2. (due 9/7) Let \( f(x) = x^3 \). Calculate the slopes of the tangent lines to the graph of \( y = x^3 \) as follows.

   (a) Let \( m_{\text{sec}} \) be the function of \( h \) that is the slope of the secant line between \( (x_0, x_0^3) \) and \( (x_0 + h, (x_0 + h)^3) \). Calculate \( m_{\text{sec}} \), obtaining the expression

   \[
   m_{\text{sec}} = (3x_0^2 + 3hx_0 + h^2) \frac{h}{h}.
   \]

   (b) In the \( h-y \) plane (horizontal coordinate \( h \) and vertical coordinate \( y \)), carefully graph the equation \( y = m_{\text{sec}} \) (completing the square to obtain \( h^2 + 3x_0h + 3x_0^2 = (h + \frac{3x_0}{2})^2 + \frac{3x_0^2}{4} \) might be a useful preliminary step). The graph will be a parabola, except that the point where the parabola meets the \( y \)-axis is missing.

   (c) If your graph is correct, the \( y \)-coordinate of the missing point is \( 3x_0^2 \). Explain, as clearly as you can, why that number is the slope of the tangent line to \( y = x^3 \) at the point \( (x_0, x_0^3) \). Clarity can be enhanced by using pictures of secant lines for some different values of \( h \).

3. (due 9/7) Let \( f(x) = x^3 \). Calculate the slopes of the tangent lines to the graph of \( y = x^3 \) as follows.

   (a) Let \( m_{\text{sec}} \) be the function of \( x \) that is the slope of the secant line between \( (x_0, x_0^3) \) and \( (x, x^3) \). Calculate \( m_{\text{sec}} \), obtaining the expression

   \[
   m_{\text{sec}} = (x^2 + xx_0 + x_0^2) \frac{x - x_0}{x - x_0}.
   \]

   (b) In the \( x-y \) plane, carefully graph the equation \( y = m_{\text{sec}} \) (completing the square to obtain \( x^2 + xx_0 + x_0^2 = (x + \frac{xx_0}{2})^2 + \frac{3x_0^2}{4} \) might be a useful preliminary step). The graph will be a parabola, except that the point where the parabola meets the line \( x = x_0 \) is missing.

   (c) If your graph is correct, the \( y \)-coordinate of the missing point is \( 3x_0^2 \). Make sure you can explain clearly why that number is the slope of the tangent line to \( y = x^3 \) at the point \( (x_0, x_0^3) \).

4. 2.2 # 1-10.

5. (due 9/7) 2.3 # 17, 19, 20, 21, 22, 24, 26, 28, 35, 36, 37, 38, 55-57.

6. (due 9/14) 2.4 # 3-6, 13, 15, 16, 20, 22-25, 26, 32, 38, 39.