19. Let \( \pi: E \to B \) be a continuous map. A **local cross-section** at \( b \) is a map \( s: U \to E \), where \( U \) is an open neighborhood of \( b \), such that \( \pi \circ s \) is the identity on \( U \), and one says that \( \pi \) has **local cross sections** if it has a local cross section at each point of \( B \). Let \( \pi: \mathcal{T}_S \to \mathbb{R}^m_{>0} \) send \( h \) to \((L_{\alpha_1}(h), \ldots, L_{\alpha_m}(h))\) as discussed in class. Prove that \( \pi \) has local cross-sections. Remark: the local product structure \( h: U \times \mathbb{R}^m \to \pi^{-1}(U) \) of \( \mathcal{T}_S \) is then defined by \( h(u, y) = y \cdot s(u) \).

20. Let \( A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \), a linear transformation of the plane \( \mathbb{R}^2 \). Find its eigenvalues \( \lambda \) and \( 1/\lambda \), where \( \lambda > 1 \), and find a pair of eigenvectors \( \{v_1, v_2\} \), for which \( v_1 \) has eigenvalue \( \lambda \) and \( v_2 \) has eigenvalue \( 1/\lambda \). Let \( e_1, e_2 \) be the standard basis, and graph the integer lattice \( \mathbb{Z}e_1 \times \mathbb{Z}e_2 \) in the \( v_1, v_2 \) basis.

21. Let \( f \) be the linear transformation of \( \mathbb{R}^2 \) which is multiplication by the matrix \( A \) in the previous problem, let \( \mu_s \) and \( \mu_u \) be the measures associated to the stable and unstable foliations associated to \( A \). Explain how the push-forward \( f \mu_u \) equals \((1/\lambda) \mu_u \).

22. Recall that \( \text{diff}(S) \) is the connected component of the identity in \( \text{Diff}(S) \). Observe that if \( g \in \text{Diff}_+(S) \) and \( h \) is a Riemannian metric on \( S \), then the push forward \( gh \) equals \( h \) if and only if \( g \) is an isometry of \( h \).

1. Our first definition of \( \mathcal{T}_S \) was the equivalence classes of hyperbolic metrics on \( S \), where \( h_1 \sim h_2 \) when there exists \( j \in \text{diff}(S) \) such that \( j h_1 = h_2 \). For this definition, the action of \( \mathcal{H}_+(S) \) on \( \mathcal{T}_S \) is \( \langle g \rangle [h] = [gh] \). Using this definition, prove that \( \langle g \rangle [h] = [h] \) if and only if \( g \) is isotopic to an isometry of \( S \) when \( S \) has the metric \( h \).

2. Our second definition of \( \mathcal{T}_S \) was the equivalence classes of marked hyperbolic structures on \( S \), that is, pairs \((S_1, g_1)\) with \( S_1 \) a surface with a hyperbolic metric \( h_1 \) and \( g_1: S_1 \to S \) is a diffeomorphism, with \((S_1, g_1) \sim (S_2, g_2)\) when \( g_2^{-1}g_1 \) is isotopic to an isometry. For this definition, the action of \( \mathcal{H}_+(S) \) on \( \mathcal{T}_S \) is \( \langle g \rangle [(S_1, g_1)] = [(S_1, gg_1)] \). Using this definition, prove that \( \langle g \rangle [(S_1, g_1)] = [(S_1, g_1)] \) if and only if \( g \) is isotopic to an isometry of \( S \), where \( S \) has the push-forward metric \( g_1 h_1 \).