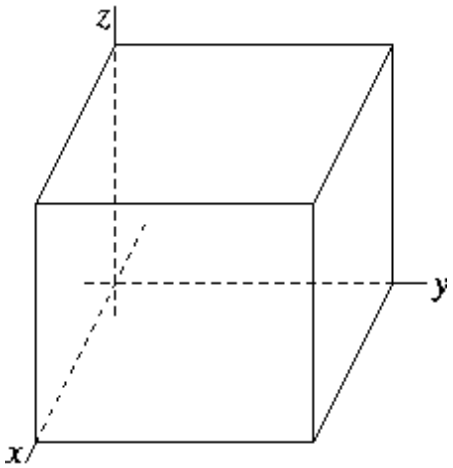
Mathematics 2433-007 Name (please print) _____

Examination III

November 21, 2002

I. (6) The figure to the right is the unit cube in xyz-space. That is, the set of points (x, y, z) for which $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$.



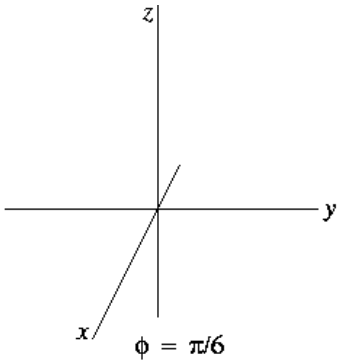
1. For the bottom face (the one in the xy-plane), draw the diagonal starting at the origin. Label it as $\vec{v}$. Then draw the diagonal of the cube, starting at the origin, and label it as $\vec{w}$.
2. Write $\vec{v}$ and $\vec{w}$ in component form.
3. Use the dot product to calculate the cosine of the angle between the two diagonals.

II. (6) Use a cross product of vectors to write an equation for the plane that contains the three points $(1, 1, 1)$, $(1, 2, 2)$, and $(2, 3, 4)$.

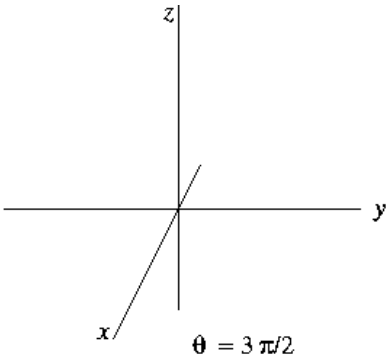
III. (6) A straight line in xyz-space has direction vector $2\vec{\imath} - 3\vec{\jmath} - \vec{k}$, and passes through the point $(1, -2, 0)$.

1. Write an equation for the line as a vector-valued function $\vec{r}(t)$.
2. Write parametric equations for the line.
3. Write equations for the line in symmetric form.

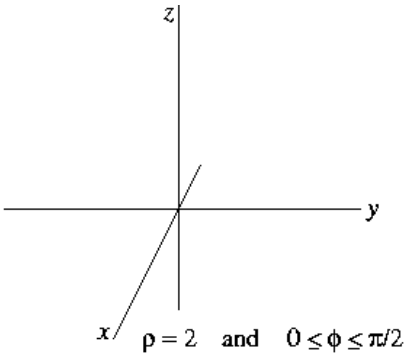
IV. (6) For each of the following equations in spherical coordinates, sketch the graph of the equation. Label values on the coordinate axes if appropriate (for example, the graph of $\rho = 1$ would be a sphere, whose intersections with the positive x, y, and z-axes would be labeled 1).



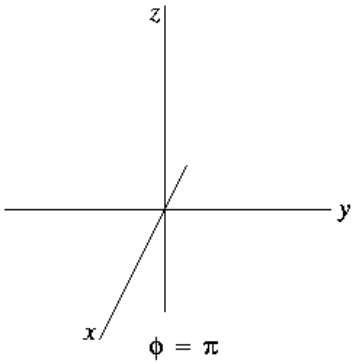
$\phi = \pi/6$



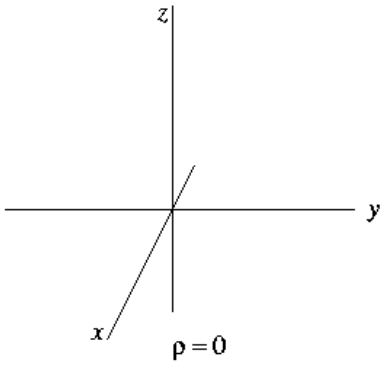
$\theta = 3\pi/2$



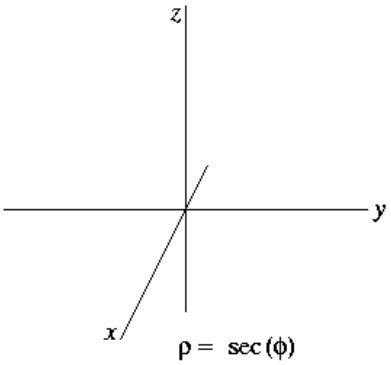
$\rho = 2$ and $0 \le \phi \le \pi/2$



$\phi = \pi$



$\rho = 0$



$\rho = \sec(\phi)$

V. (6) Let $\vec{a} = \vec{\imath} + \vec{\jmath} - 3\vec{k}$.

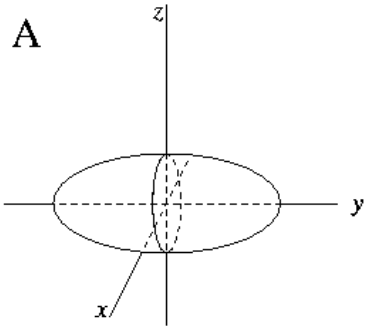
1. Find a unit vector in the direction of $\vec{a}$.

2. Determine whether $\vec{a}$ is perpendicular to $3\vec{\imath} + 3\vec{\jmath} + 2\vec{k}$.

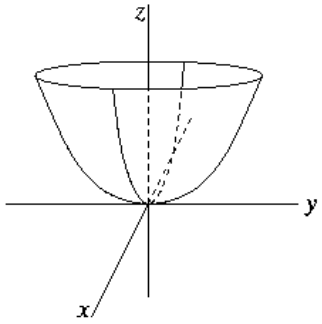
3. Find the scalar projection of $7\vec{\imath}$ to $\vec{a}$.

VI. (6) The figure below shows six quadric surfaces, labeled A through F. Answer the questions below by giving the correct letter or letters.

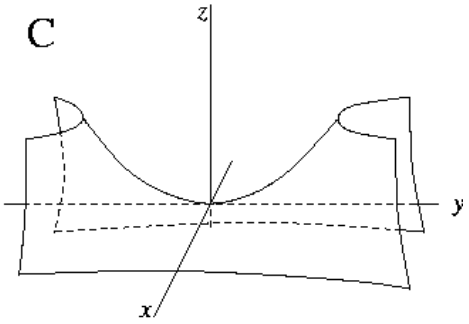
A



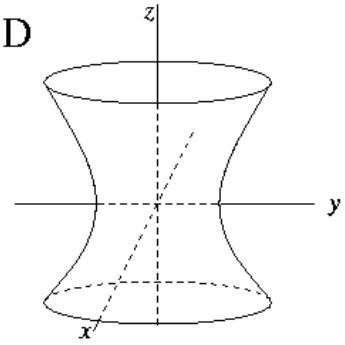
B



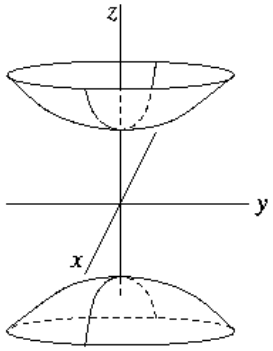
C



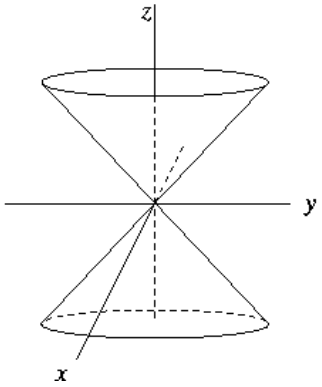
D



E

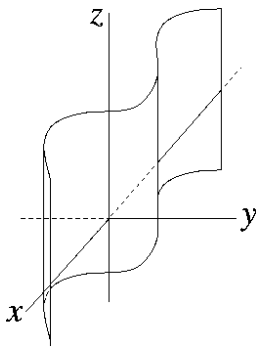


F

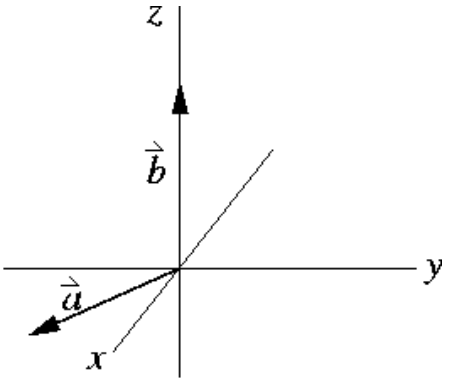


1. Which surface or surfaces have horizontal cross-sections that are ellipses or circles?
2. Which surface has an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$?
3. Which surface or surfaces have cross-sections perpendicular to the y-axis that are hyperbolas?
4. Which surface has an equation of the form $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$?
5. Which surface looks most like the graph of $z = y^2 - x^2$?
6. Which surface or surfaces have at least one trace that is a parabola?

VII. (3) The figure to the right shows a surface which is made up of vertical lines. The surface contains the z-axis. Give a possible equation for the surface.



VIII. (6) The figure to the right shows two vectors $\vec{a}$ and $\vec{b}$ that start at the origin. The vector $\vec{a}$ lies in the xy-plane, while the vector $\vec{b}$ points along the z-axis.



1. Tell why $\vec{a} \times \vec{b}$ lies in the xy-plane (assuming that it is located to start at the origin).

2. In the diagram to the right, draw a vector in the direction of $\vec{a} \times \vec{b}$ (locate it starting at the origin).

3. If $\| \vec{a} \| = 3$ and $\| \vec{b} \| = 4$, what is $\| \vec{a} \times \vec{b} \|$?

IX. (4) A certain straight line has parametric equations $x = 3 + 3t$, $y = 2 - 3t$, and $z = t$.

1. Write an equation for the plane that passes through the point $(2, 2, -1)$ and has normal vector $\vec{\imath} - \vec{\jmath} + \vec{k}$.

2. At what point do this line and this plane intersect?

X. (5) For each of the following, tell whether the expression is a scalar (i. e. a number), a vector, or is meaningless.

1. $\| \vec{a} \times \vec{b} \|$
2. $(\vec{a} \cdot \vec{b})\ \ \vec{a} \times \vec{b}$
3. $\dfrac{\vec{a} \times \vec{b}}{\vec{b} \times \vec{b}}$
4. $\dfrac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}$
5. $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$