Mathematics 2433-007 Name (please print) ____________________

Examination II

October 24, 2002

I. For the series $\sum_{n=1}^{\infty} a_n$:
(6)

1. Define the n^{th} *partial sum* s_n.

2. Define what it means to say that the series *converges.*

3. Suppose that $s_n = n^2$. Calculate a_n.

II. Verify that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges, using the following two methods:
(6)

1. By using the Limit Comparison Test to check that it has the same convergence behavior as $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

2. By using the Comparison Test with a direct comparison to the terms of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

III. (6) For each of the following functions, write the Maclaurin series both as a summation and as an infinite list of terms. For example: $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

1. e^x

2. $\cos(x)$

IV. (16) For each of the following power series, determine the convergence behavior. That is, find the interval of convergence, and for the x in the interval of convergence, tell where the convergence is conditional and where it is absolute. Follow any special instructions given.

1. $\sum_{n=2}^{\infty} \frac{x^n}{(\ln(n))^n}$ (use the Root Test).

2. $\sum_{n=2}^{\infty} (\ln(n))^n x^n$

3. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$ (use the Ratio Test).

V. (6) The Alternating Series Test does not apply to any of the following series. For each of the series, tell why the Alternating Series Test does not apply.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^2}{n^2}$

2. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{(-2)^n}$

3. $\displaystyle\sum_{n=1}^{\infty} (-1)^n b_n$, where $b_n = \dfrac{1}{n}$ if n is even and $b_n = \dfrac{1}{n^2}$ if n is odd.

VI. (7) Write the general formula for the Taylor series of $f(x)$ at $x = a$. Use the formula with $a = 1$ to calculate the Taylor series of $\dfrac{1}{x}$ at $a = 1$.

VII. Suppose that a function $f(x)$ can be written as $c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$ for all x in some interval that
(5) contains 0.

1. Show that $c_0 = f(0)$.

2. Write a series, involving these c_n, for $f'(x)$, and use it to find c_1.

3. Write a series, involving these c_n, for $f''(x)$, and use it to find c_2.

4. Write a series, involving these c_n, for $f^{(3)}(x)$, and use it to find c_3.

5. Write a general expression for c_n.

VIII. Show that if $0 < b_n < \dfrac{1}{n}$ for all n, then $\displaystyle\sum_{n=1}^{\infty} \frac{b_n}{n}$ converges.
(4)