Mathematics 2433-007 Name (please print) ______________________

Examination I

September 19, 2002

I. Consider the polar equation $r = 1 + \cos(\theta)$.
(5)

1. In a (θ, y)-coordinate system, sketch the graph of the Cartesian equation $y = 1 + \cos(\theta)$.

2. In an (x, y)-coordinate system, sketch the graph of the polar equation $r = 1 + \cos(\theta)$.

II. Using $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$, calculate the general formula for $\dfrac{dy}{dx}$ for a polar curve in which r is a function of θ.
(4)

III. For the parametric equations $x = e^t$, $y = -e^{2t}$, solve for t in terms of x and then eliminate the parameter t to obtain an xy-equation. On a graph, show exactly where and in what direction a point P with coordinates $(e^t, -e^{2t})$ moves as t goes from $-\infty$ to ∞. Hint: to simplify the xy-equation, you can use the fact that $a\ln(b) = \ln(b^a)$.
(6)

IV. (6+) On the ellipse $x = 4\cos(t)$, $y = 3\sin(t)$, there are two points where the tangent line has slope 1.

1. Calculate an expression for $\frac{dy}{dx}$ in terms of t.

2. Using the inverse tangent function, find a value of t between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ where the tangent line has slope 1 (the answer should contain $\tan^{-1}(r)$ for some number r).

3. Find the exact x and y-coordinates where that tangent line occurs on the ellipse (the answer can contain $\tan^{-1}(r)$ for some number r).

4. (For up to 3 bonus points) Using triangles or a diagram in the xy-plane, simplify the expressions for these x and y-coordinates to express their exact values without using the inverse tangent function.

V. (3) Calculate the value of the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{e^{2n}}$.

VI. Consider the line segment $x = t$, $y = 1 - t$, $0 \le t \le 1$. It runs from $(0, 1)$ to $(1, 0)$.
(6)

1. Use ds and an integral to calculate that the length of this segment is $\sqrt{2}$.

2. Draw a picture of the cone obtained when this line segment is rotated around the x-axis. Use ds and an integral to calculate the surface area of this cone.

VII. Consider the line segment $x + y = 1$, $0 \le x \le 1$.
(4)

1. Change the equation $x + y = 1$ to polar coordinates, and solve for r in terms of θ.

2. Write an integral in terms of θ whose value is the area of the triangle bounded by the line segment and the x and y axes. Supply the limits of integration, but *do not* try to calculate the value of the integral.

VIII. Give the mathematical definition of what it means to say that the series $\sum_{n=1}^{\infty} a_n$ *convereges* to L.
(3)

IX. (10) For each of the following sequences, find the limit, if the sequence converges. If the sequence diverges, determine whether it diverges to ∞, or $-\infty$, or neither. *Give some explanation or indication of your reasoning, beyond just finding values on a calculator.* In particular, when the series is one of the basic types considered in class, such as n^p or r^n, note this and use your knowledge of the behavior of those sequences to determine the limit.

1. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{1+n}$

2. $\lim\limits_{n\to\infty} \dfrac{1}{(0.99999999999999739)^n}$

3. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{1+\sqrt{n}}$

4. $\lim\limits_{n\to\infty} n^{-0.00000000000000001}$

5. $\lim\limits_{n\to\infty} \arctan(n/2)$

X. (3) Explain how one knows immediately that the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2n+3n^2}$ diverges.