Mathematics 2433-001 | Name (please print) ______________________

Examination III Form A | Student Number ____________

November 29, 1999

I. Use the Maclaurin series $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$ to find the Maclaurin series for $x\sin\left(\frac{x}{2}\right)$.
(3)

II. For the points $P = (1, 2, 3)$, $Q = (0, 3, 7)$, and $R = (3, 5, 11)$:
(11)

1. Write the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ in the form $a\vec{\imath} + b\vec{\jmath} + c\vec{k}$.

2. Calculate the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

3. Calculate $\sin(\theta)$, where θ is the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

4. Write an equation for the plane that contains P, Q, and R.

III. (3) Calculate $\vec{a} \cdot \vec{b}$, where $\| \vec{a} \| = 3$, $\| \vec{b} \| = 4$, and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{5\pi}{6}$.

IV. (3) Find all values of x for which the vectors $x\vec{\imath} + 3x\vec{\jmath} - \vec{k}$ and $x\vec{\imath} - \vec{\jmath} + 4\vec{k}$ are orthogonal.

V. (4) Suppose that a power series $\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$ is equal to $f(x)$ for all x. Notice that $f(a) = c_0 + c_1 \cdot 0 + c_2 \cdot 0 + \cdots = c_0$. Express $f'(x)$ and $f''(x)$ as power series centered at $x = a$, and use these series to calculate c_1 and c_2.

VI. (4) Find a unit vector $\vec{u}$ in the direction of $\vec{\imath} - 2\vec{\jmath}$. Write parametric equations for a point moving at unit speed along a straight line, with velocity vector $\vec{u}$, if the point is at $(6, 1, 7)$ when $t = 0$.

VII. (8) For the equation $x^2 + y^2 - z^2 = 1$, calculate the traces for $z = k$ and for $y = k$. Sketch the traces for $z = k$ in the xy-plane, and tell what kind of curves are the traces for $y = k$ in the xz-plane (but do not take time to graph them carefully). This tells you what kind of quadric surface the equation represents. What is it called? Make a rough sketch of it.

VIII. Consider the vector-valued function $\vec{r}(t) = \vec{\imath} + \tan(t)\vec{\jmath} + \sec(t)\vec{k}$.
(8)

1. Calculate $\vec{r}\,'(t)$.

2. Write an equation *as a vector-valued function* for the tangent line to the curve represented by $\vec{r}(t)$ at the point $(1, 1, \sqrt{2})$.

3. Write a definite integral whose value is the length of the portion of this curve that runs from $(1, 0, 1)$ to $(1, 1, \sqrt{2})$, but do *not* try to evaluate the integral.

IX. In this problem, all vectors are assumed to start at the origin. Draw two xyz-coordinate systems. On the
(6) first, draw the vectors $\vec{\imath}$, $-2\vec{k}$, and a typical vector $\vec{v}$ of length between 2 and 3 that lies in the plane $x = 0$ and has positive y- and z-components. Draw and label the cross products $\vec{v} \times \vec{\imath}$ and $\vec{v} \times (-2\vec{k})$. On the second coordinate system, draw $\vec{v}$ and $-2\vec{k}$, and draw and label the vector projection of $\vec{v}$ onto $-2\vec{k}$ and the vector projection of $-2\vec{k}$ onto $\vec{v}$.