Mathematics 2433-001 | Name (please print) ____________________
Examination II Form A | Student Number ______________
October 22, 1999

I. For each of the following power series, use one of the following tests to determine convergence or divergence:
(15) Integral Test, Comparison Test, Alternating Series Test, Ratio Test.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$

II. For each of the following, circle the letter of the correct response.
(15)

1. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

(a) all values (b) $p < 0$ (c) $p \leq 0$

(d) $p < 1$ (e) $p \leq 1$ (f) $p > 0$

(f) $p \geq 0$ (g) $p > 1$ (i) $p \geq 1$

2. To show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$ converges, one could compare its terms to the terms of which of the following series?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^2}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \arctan(n+1)$ (e) $\displaystyle\sum_{n=1}^{\infty} \ln(n^2+2n+2)$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4+n^3+2}$ (h) more than one of these (i) none of these

3. The series $\displaystyle\sum_{n=0}^{\infty} 2^n x^{2n}$ converges for

(a) only $x = 0$ (b) $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

(d) $-\frac{1}{2} < x < \frac{1}{2}$ (e) $-\frac{1}{2} \leq x \leq \frac{1}{2}$ (f) $-1 < x < 1$

(g) $-1 \leq x \leq 1$ (h) $-\sqrt{2} < x < \sqrt{2}$ (i) $-\sqrt{2} \leq x \leq \sqrt{2}$

(j) $-2 < x < 2$ (k) $-2 \leq x \leq 2$ (l) $-\infty < x < \infty$

III. (5) A power series of the form $\sum_{n=0}^{\infty} c_n(x-3)^n$ has radius of convergence $R = 5$. What can be said about its convergence or divergence at different values of x?

IV. (7) The Maclaurin series for $\sin(x)$ is $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

(a) Use the ratio test to verify that this series converges absolutely for all x.

(b) Differentiate this series term by term to obtain a power series for $\cos(x)$.

V. (8) Starting with the fact that

$$\ln(1+x) = \int \frac{1}{1+x}\,dx = \int \frac{1}{1-(-x)}\,dx\ ,$$

expand $\dfrac{1}{1-(-x)}$ as a power series, and integrate to obtain a power series, plus a constant, which is equal to $\ln(1+x)$. Evaluate at $x = 0$ to show that the constant equals 0.

VI. (6) Define *absolutely convergent* and *conditionally convergent.* Give an example of a conditionally convergent series.