Mathematics 2443-001 Name (please print) ______________________

Test 1 Form A

September 24, 1999

I.(12) A particle moves with position (x, y) where $x = \sin(t)$ and $y = \csc^2(t)$ for $\frac{\pi}{6} \leq t \leq 1$.

a. On a graph, show the path of the particle and describe the motion.

b. Write an integral whose value is the distance traveled by the particle, but *do not* simplify or try to evaluate the integral. (Recall that $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$.)

c. Suppose that the path followed by the particle is rotated around the line $y = -2$ to form a surface. Write an integral whose value is surface area of the resulting surface, but *do not* simplify or try to evaluate the integral.

II.(6) Find *all* polar coordinates for the point with xy-coordinates $(-1, 7)$. (An inverse tangent should appear in your answer— do not replace it by a decimal approximation.)

III.(6) Making use of the chain rule $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$, calculate a formula for $\frac{dy}{dx}$ in terms of $\frac{dr}{d\theta}$.

IV.(10) For the equation $r = 3\sin(2\theta)$, graph the equation in the $r\theta$-plane, and also in the xy-plane where r and θ represent polar coordinates. Calculate the area enclosed by one loop of the polar curve. (Hint: $\sin^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}$.)

V.(4) For the series $\sum_{n=1}^{\infty}(-1)^n n^2$, calculate the partial sums s_1, s_2, and s_4.

VI.(4) Explain how you can know that the series $\sum_{n=1}^{\infty} \sin\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ diverges.

VII.(4) Give the exact value of the series $\sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{4}\right)^n$.

VIII.(4) Find the values of x for which the following series converges: $\sum_{n=0}^{\infty} (-1)^n (x-2)^n$.

IX.(4) Bonus: Analyze the behavior of the polar curve $r = \frac{1}{\theta^2}$ as $\theta \longrightarrow 0^+$.