Topology II Midterm
March 15, 2019

Instructions: *In writing your solutions be sure to make it clear what the relevant basic definitions are. When appropriate you may refer to results that have been established in class. Completing five problems correctly will earn a score of 86/100.*

PROBLEM 1. (a) What is a retract? Show that if A is a retract of X then the inclusion map $A \to X$ induces a one-to-one homomorphism on fundamental groups.
(b) Show that projection onto the first factor of a nonempty product space $X \times Y$ is a retraction but that it may not be a deformation retraction.

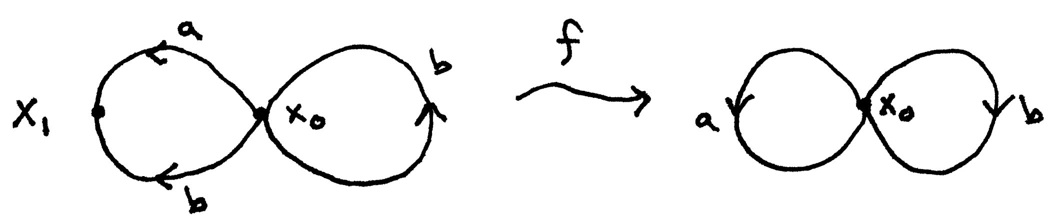
PROBLEM 2. Show that a compact metric space is totally bounded but that a complete metric space may not be totally bounded.

PROBLEM 3. Let X be a space and $x_0, x_1 \in X$. Give a formula for a path homotopy that shows that the concatenation $f \cdot \overline{f}$ of a path from x_0 to x_1 with its reverse is path homotopic to the constant path c_{x_0}.

PROBLEM 4. Identify the fundamental group of each space up to isomorphism. No explanations needed: (a) $S^1 \times \mathbb{R}^2$ (b) $S^2 \times S^3$ (c) $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1 \text{ and } x^2 + y^2 \neq 0\}$ (d) $S^1 \times S^1 - \{point\}$.

PROBLEM 5. Let $g : S^1 \to S^1$ be the homeomorphism given by conjugation (where $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$. Show that g is not homotopic to the identity function on S^1.

PROBLEM 6. Let $X = S^1 \vee S^1$. The picture below describes a continuous surjection $f : X \to X$ where $f(x_0) = f(x_1) = x_0$ and each open labeled segment in the domain gets sent homeomorphically to its counterpart with the same label in the codomain. Show that $f_* : \pi_1(X, x_0) \to \pi_1(X, x_0)$ is an isomorphism.



PROBLEM 7. Show that any two continuous functions from X to Y are homotopic if Y is contractible.