Math 5863
Assignment # 1
due: Friday January 25

PROBLEM 1. Show that in a metric space (X, d) if (x_n) is a Cauchy sequence then $\{x_n \mid n \in \mathbb{Z}_+\}$ is bounded. (A subset of X is *bounded* iff it is contained in $B(x, N)$ for some $x \in X$ and $N \in \mathbb{Z}_+$.)

PROBLEM 2. Use the existence of a space-filling curve to show that for each positive integer n there is a continuous function from $I = [0, 1]$ onto I^n.

PROBLEM 3. Let X be a topological space and let Y be a dense subset of X. Show that if A is a nowhere dense subset of Y (in the subspace topology on Y) then A is nowhere dense in X. (A subset of a topological space is *nowhere dense* iff its closure has empty interior.)
Hint: You might want to recall theorem 17.4.