Math 5863
TF Problem Set # 5
due Wednesday, March 6.

*Instructions*: Determine whether each statement is true or false and be prepared to orally provide a brief proof or counterexample supporting your conclusion. This is a group assignment in which you must consult with classmates, comparing answers before the due date!

PROBLEM 1. *If $f, g : X \to Y$ are homotopic maps and $A \subset X$ then $f|_A$ is homotopic to $g|_A$.*

PROBLEM 2. *$\partial B^2$ is a retract of $B^2$.*
*(Recall: $B^2 = \{z \in \mathbb{C} \mid |z| \leq 1\}$ and $\partial B^2 = S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$.)*

PROBLEM 3. *The fundamental group of $\mathbb{R}^3$ with the $z$-axis removed is infinite cyclic. ($z$-axis equals $\{0\} \times \{0\} \times \mathbb{R}$.)*

PROBLEM 4. *The fundmental group of $\mathbb{R}^2 - \{(0, \pm 1), (\pm 1, 0)\}$ is abelian.*

PROBLEM 5. *A retract of a simply connected space is simply connected.*

PROBLEM 6. *A retract of a space that is not simply connected space is not simply connected.*

PROBLEM 7. *There is a homeomorphism $f : B^2 \to B^2$ with $f(0) \in \partial B^2$.*

PROBLEM 8. *The wedge of two path connected spaces is path connected.*

PROBLEM 9. *Let $X = \{a, b, c, d\}$ be the topological space with basis $\mathcal{B} = \{\{a\}, \{b\}, \{a, b, c\}, \{a, b, d\}\}$. The function $H : I \times I \to X$ defined below is a path homotopy.*

$$H(s,t) = \begin{cases} a & \text{if } t > s \text{ and } t > 1 - s \\ c & \text{if } t \leq s \text{ or } t \leq 1 - s \end{cases}$$

PROBLEM 10. *The topological space $X$ from problem 9 is path connected.*

PROBLEM 11. *The topological space $X$ from problem 9 is simply connected.*