

Math 5863

TF Problem Set # 4

due Wenesday, February 20.

Instructions: Determine whether each statement is true or false and be prepared to orally provide a brief proof or counterexample supporting your conclusion. This is a group assignment in which you must consult with classmates, comparing answers before the due date!

PROBLEM 1. For every topological space X the empty subset is a deformation retract of X .

PROBLEM 2. The singleton set consisting of the origin is a deformation retract of \mathbb{R}^2 with the topology induced by the French railway metric.

PROBLEM 3. Let X and Y be topological spaces with $y_0 \in Y$. The (projection) map $p_1(x, y) = (x, y_0)$ from $X \times Y$ to $Y \times \{y_0\}$ is a retraction.

PROBLEM 4. In the previous problem if $Y = I^2$ then p_1 is a deformation retraction.

PROBLEM 5. If $f : I \rightarrow X$ is a path from x_0 to x_0 in X then f is null-homotopic.

PROBLEM 6. A surface with odd Euler characteristic is non-orientable.

PROBLEM 7. The surfaces with id patterns $aba^{-1}b$ and c^2d^2 are homeomorphic.

PROBLEM 8. The 2-disk with identification id $abcac^{-1}ab$ is a nonorientable surface with genus 3.

PROBLEM 9. The 2-disk with id pattern $abcac^{-1}ab$ has Euler characteristic -1 .

PROBLEM 10. There is a 2-disk with id pattern X representing the torus T^2 for which radial projection gives a deformation retraction from $X - \{\text{origin}\}$ onto a subspace of X homeomorphic to $\{(x, 0) \mid -2 \leq x \leq 2\} \cup \{(0, y) \mid 1 \leq y \leq 2\} \cup \{(x, y) \mid x^2 + y^2 = 1\}$.

PROBLEM 11. If X is a contractible space then $\{x_0\}$ is a deformation retract of X for some $x_0 \in X$.