Topology I Final Exam
December 11, 2018

PART I:

1. Let a and b be real numbers with $a < b$ and consider the closed interval $[a, b]$ with the Euclidean topology. Prove one of the following:
(a) $[a, b]$ is compact.
(b) $[a, b]$ is connected.

2. Work one of the following:
(a) Let $\mathcal{C}$ be a set whose elements are continuous functions from a space X to $I = [0, 1]$ (with Euclidean topology). Describe the evaluation map $e : X \to I^{\mathcal{C}}$ and show that it is continuous.
(b) Give an outline of the proof that the product of two connected spaces is connected.

PART II:

3. Show that subspaces of regular spaces are regular.

4. Let $f : X \to Y$ be a function between spaces X and Y. Show that f is continuous if $f^{-1}(C)$ is closed in X whenever C is closed in Y.

5. Show that the cofinite topology on any set X is compact.

6. Let X be a space and $A \subseteq X$.
(a) Let $\mathcal{B}$ be a basis for the topology on X. Show that A is dense in X if every $B \in \mathcal{B}$ contains an element of A.
(b) Is the statement in (a) true if the basis $\mathcal{B}$ is replaced with a subbasis $\mathcal{S}$? Explain.

7. Let X and Y be Hausdorff topological spaces and suppose that $f : X \to Y$ and $g : X \to Y$ are continuous. Show that the set $A = \{x \in X \mid f(x) = g(x)\}$ is closed.

PART III: State whether each of the following is true or false and justify your answer.

8. A space X which is compact, Hausdorff and second countable is metrizable.

9. Every separable first countable space is second countable.

10. With the cofinite topology $\mathbb{R}$ is path connected.

11. If (X, d) is a metric space, $x \in X$ and $\epsilon > 0$ then the closure of $B(x, \epsilon)$ equals $\{y \in X \mid d(x, y) \leq \epsilon\}$.

12. Let $\mathcal{T}$ be the topology on $\mathbb{R}$ generated by the basis $\mathcal{B} = \{(a, \infty) \mid a \in \mathbb{R}\}$. A continuous function from $(\mathbb{R}, \mathcal{T})$ to $\mathbb{R}_\ell$ must be constant.