

**Project 3, Part I:** Consider the following statement and its "proof".

The first sentence asserts two things:  
① There is a largest positive integer.  
② Call that largest positive integer  $n$ .  
There is no basis for knowing that ① is true. So the rest of the argument is invalid.

In other words, ① is a false assumption.

CLAIM 1. *There is no integer larger than 1.*

*Proof.* Let  $n$  be the largest positive integer. Since every positive integer is greater or equal than 1, we know that  $n \geq 1$ . Multiplying both sides of this inequality by the positive integer  $n$  shows that  $n^2 \geq n$ . However we also know that  $n \geq n^2$  because by assumption  $n$  is the largest positive integer. From these two inequalities we conclude that  $n^2$  and  $n$  must be equal. Now dividing both sides of the equation  $n^2 = n$  by  $n$  (which is positive and therefore not equal to 0) we obtain that  $n = 1$ . So 1 is the largest integer which means that there is no integer larger than 1.  $\square$

If the first sentence had instead said:  
"Suppose that there is a largest integer  $n$ ."  
Then the rest of the argument would be valid.  
Since the conclusion that "there is no integer larger than 1" is false we conclude that the supposition that "there is a largest integer" is false.

Take away: "Assumptions" are not the same as "suppositions". An assumption is assumed to be true; a supposition may or may not be true.