

Example How many surjections from $X = \{1, 2, 3, 4\}$ to $Y = \{a, b, c\}$? $81 - 45 = 36$

Note There are 0 injections from X to Y .
but there are lots of surjections.

$$|\{f: X \rightarrow Y\}| = 3^4 = 81$$

Let's count functions which are not surjections.

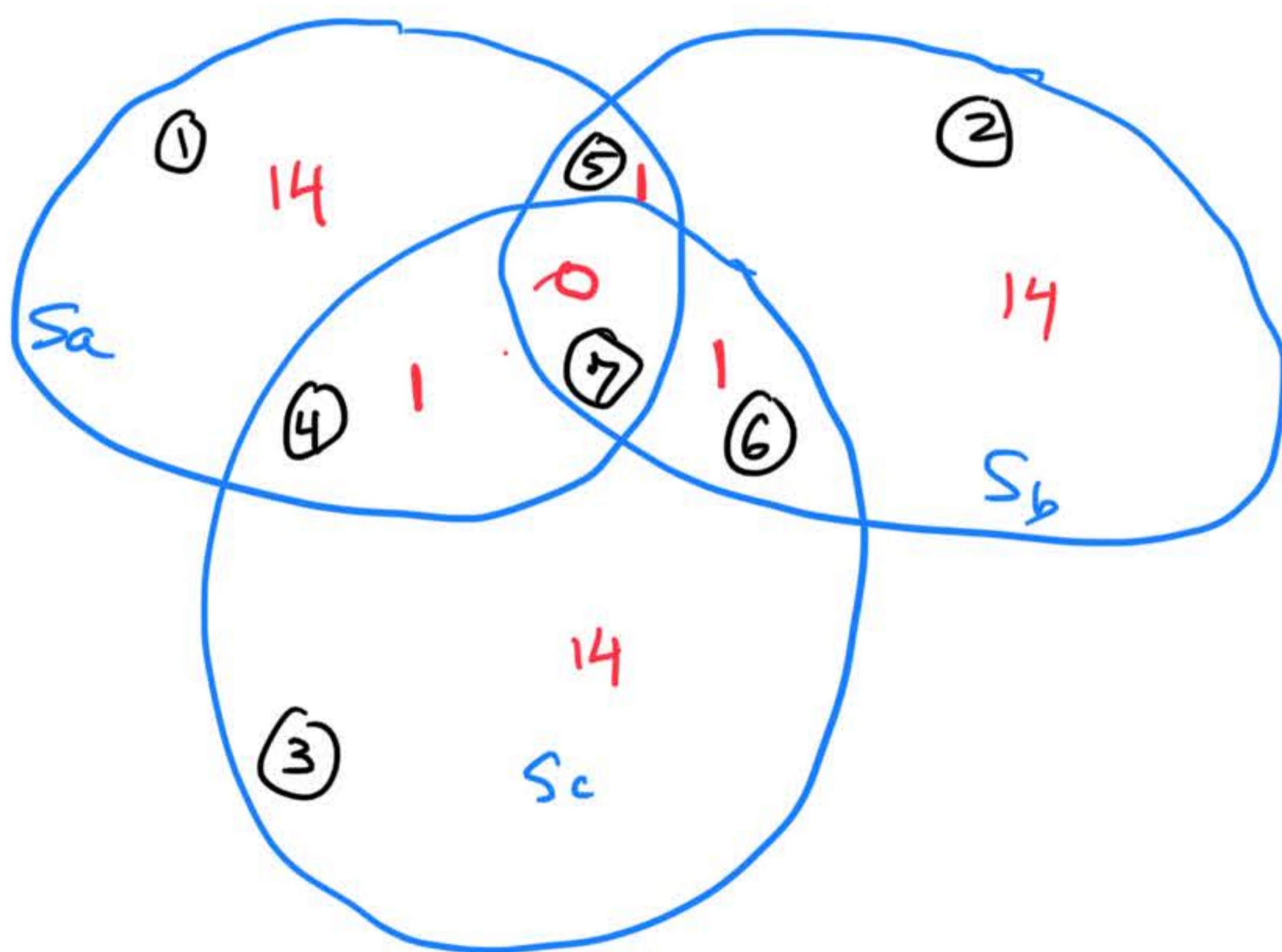
If $f: X \rightarrow Y$ is not surjection then either $a \notin f(X)$ or $b \notin f(X)$ or $c \notin f(X)$.

$$S_a = \{f: X \rightarrow Y \mid a \notin f(X)\}$$

$$S_b = \{f: X \rightarrow Y \mid b \notin f(X)\}$$

$$S_c = \{f: X \rightarrow Y \mid c \notin f(X)\}$$

We want to determine $|S_a \cup S_b \cup S_c|$.



$$\begin{aligned} &= 3 \cdot 14 + 3 - 1 \\ &= 45 \end{aligned}$$

$$|S_a| = |\{f: X \rightarrow Y - \{a\} = \{b, c\}\}| = 2^4 = 16$$

$$|S_b| = 16$$

$$|S_c| = 16 = \textcircled{3} + \textcircled{4} + \textcircled{7} + \textcircled{6} \Rightarrow \textcircled{3} = 14$$

$$|S_a \cap S_b| = |\{f: X \rightarrow \{c\}\}| = 1^4 = 1$$

$$|S_a \cap S_c| = 1$$

$$|S_b \cap S_c| = 1 = \textcircled{6} + \textcircled{7} = \textcircled{6}$$

$$|S_a \cap S_b \cap S_c| = 0 = \textcircled{7}$$