$a_0 = 0, a_1 = 1$

~~$a_{n+2} = a_n + a_{n+1}$~~

$$a_{n+2} = 5a_n + 3a_{n+1} \quad (*)$$

← recursion only
no initial conditions

Does $a_n = r^n$ for some $r \in \mathbb{R}$ satisfy $(*)$?

$$a_{n+2} = r^{n+2}, \quad a_{n+1} = r^{n+1}$$

So $(*)$ $$r^{n+2} = 5r^n + 3r^{n+1}$$ ←

Divide by r^n, $$r^2 = 5 + 3r$$

← quadratic equation

$$r^2 - 3r - 5 = 0$$

$$r = \frac{3 \pm \sqrt{29}}{2}$$

$b^2 - 4ac =$
$9 + 20$

Yes

$$a_n = \left(\frac{3+\sqrt{29}}{2}\right)^n \quad \text{solves } (*)$$

$$a_0 = \left(\frac{3+\sqrt{29}}{2}\right)^0 = 1$$

$$a_1 = \frac{3+\sqrt{29}}{2}$$

) ← initial conditions

$$a_n = \left(\frac{3-\sqrt{29}}{2}\right)^n \quad \text{solves } (*)$$