

$m = 14$ Inverses?

$\mathbb{Z}_{14} = \{0, 1, 2, 3, \dots, 13\}$

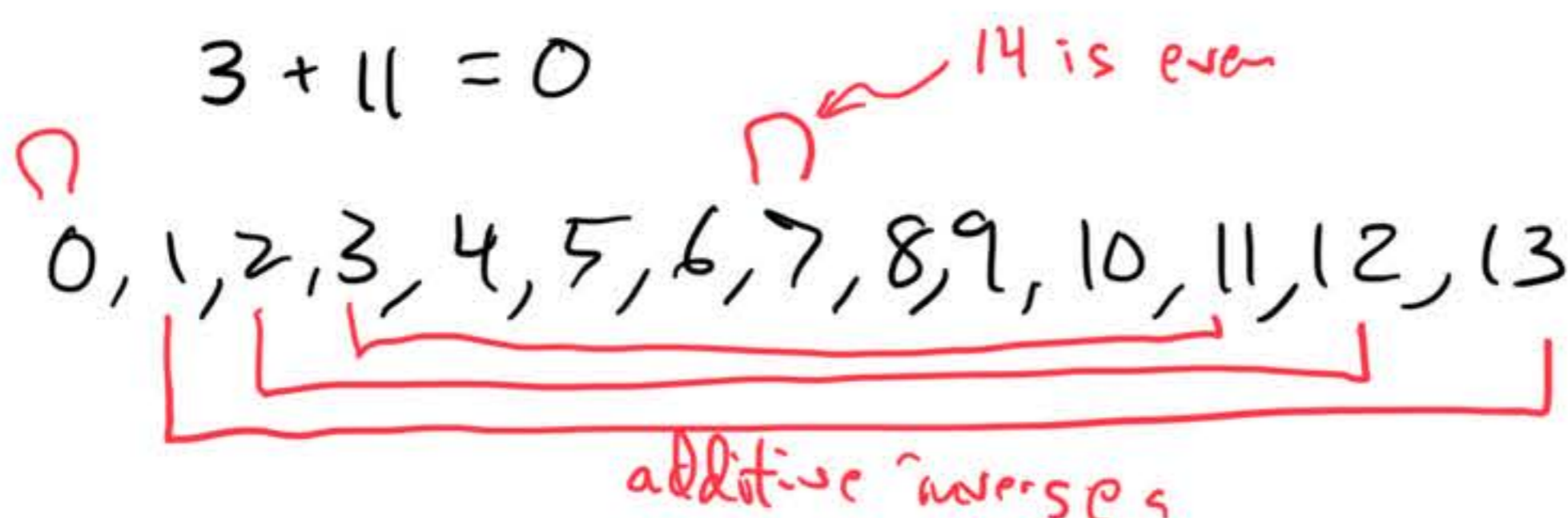
$49 = 3 \cdot 14 + 7$

$7+7 = 7^2 = 7$ example

additive inverse of a would be $-a$,
 $14-a \equiv_{14} -a$ So additive inverse of a is $14-a$.

check $a + (14-a) = 0$
 ↑
 between 0 and 13

$3 + 11 = 0$



$-13 = 1$
 $13 = -1$

$7+7=0, 0+0=0$

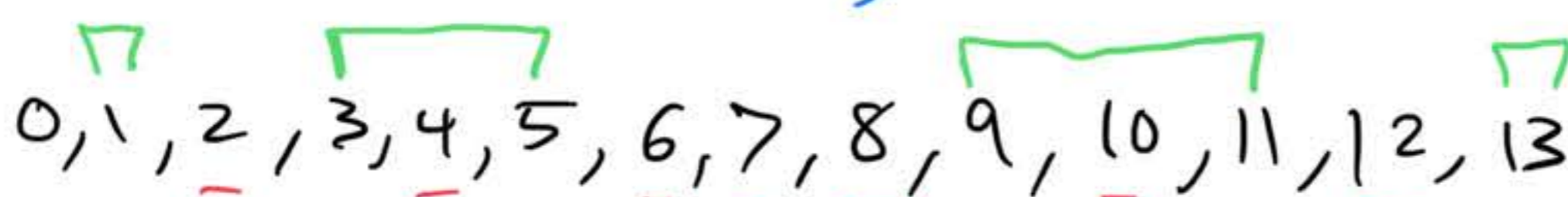
multiplication in \mathbb{Z}_{14}

$1 \times 1 = 1$

$3 \times 5 = 1$ ← 3 and 5 are mult. inverses.

"
 5×3

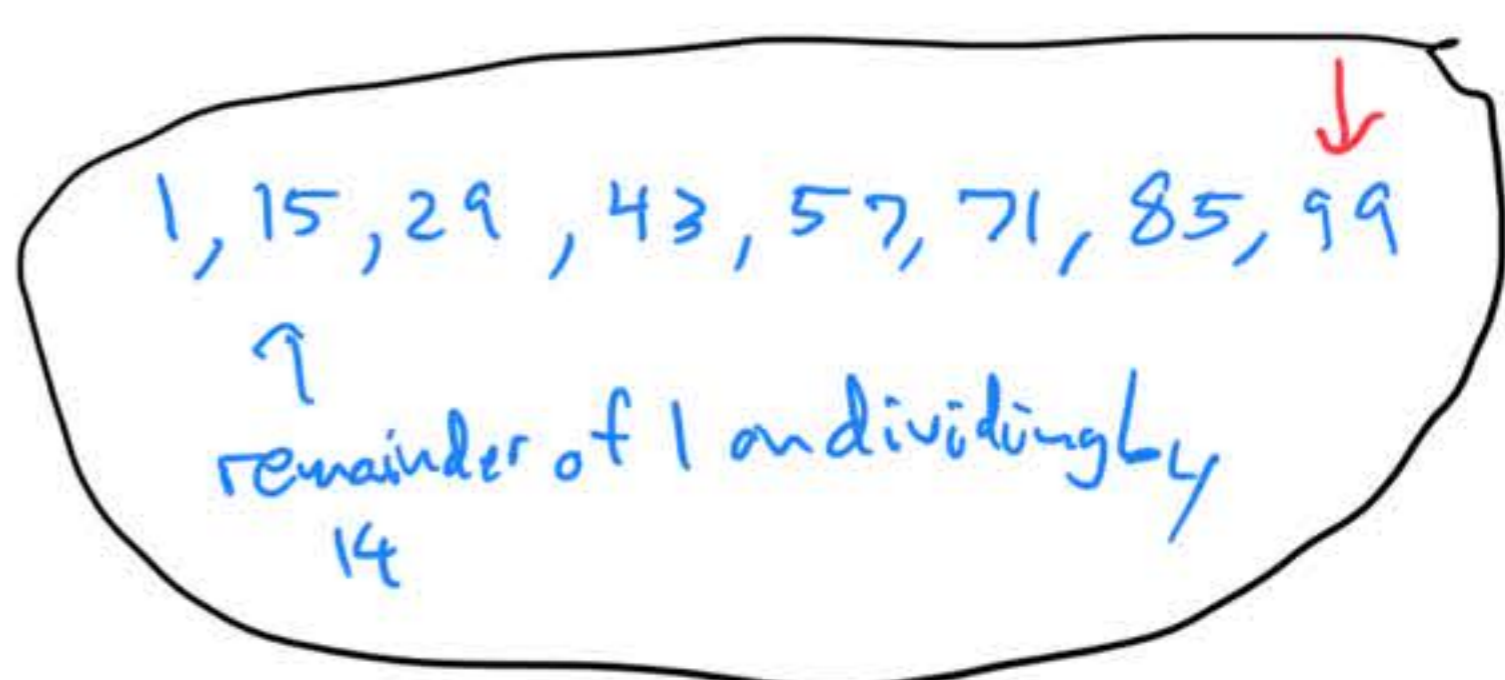
" $\frac{1}{3} = 5$ in \mathbb{Z}_{14} "



$9 \times 11 = 1$

$13 \times 13 = 1$

"
 $(-1) \times (-1) = 1$



$12 = 2 \times 6$ $\text{gcd}(12, 14) = 2$

conclude 12 has no m.i. ← try mult. 12 times everything.

$7 \times 12 = 14 \cdot 6 = 0$

Suppose $12a = 1$ then $7 \times (12a) = 7$

but $7 \neq 0$ in \mathbb{Z}_{14}

"
 $(7 \times 12) \times a$
 "
 $0 \times a = 0$

a	12x a
0	0
1	12
2	10
3	8
4	6
5	4
	⋮
	⋮

$36 = 2 \times 14 + 8$

← look for 1's

60
 $56 + 4$

In \mathbb{Z}_5 the equation $x^2 + 1 = 0$ has a solution.

$2^2 + 1 = 0$

x	$x^2 + 1$
0	1
1	2
2	0
3	0
4	2

← $x^2 + 1 = 0$ has 2 solutions.