Principle of Inclusion/Exclusion

- For two finite sets A_1 and A_2:

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

- For three sets A_1, A_2, A_3:

used this previously

$$|A_1 \cup A_2 \cup A_3|$$
$$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$
$$= \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k|$$
$$= \sum_{1 \le i \le 3} |A_i| - \sum_{1 \le i < j \le 3} |A_i \cap A_j| + \sum_{1 \le i < j < k \le 3} |A_i \cap A_j \cap A_k|$$

- Extends to $|A_1 \cup A_2 \cup \cdots \cup A_n|$ ....

three sets A_1, A_2, A_3

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

First count $|A_1| + |A_2| + |A_3|$, but observe that each element of $A_1 \cap A_2$, $A_1 \cap A_3$, $A_2 \cap A_3$ will be counted twice. Then compensate by counting $|A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3|$. Finally, observe that now none of the elements of $A_1 \cap A_2 \cap A_3$ have been counted, so add $|A_1 \cap A_2 \cap A_3|$.