More
Exam 2 Review

#4(d). In $\mathbb{Z}_{15}$, solve for x if $7x+b=0$.

$13 \cdot 7 = 1$ in $\mathbb{Z}_{15}$, $-13 = 2$ in $\mathbb{Z}_{15}$

$7x+b=0 \Rightarrow 7x=-b \Rightarrow 13(7x) = 13(-b)$

$13(7x) = (13 \cdot 7)x = 1 \cdot x = x$, $13(-b) = (-13)b = 2b$

answer $x = 2b$

#5. $A = \{1,2,3,\dots,9\} = [1,9]$

(a) How many subsets with four elements?

(b) How many subsets with four elements contain "5"?

(c) How many subsets have four elements but don't contain both 5 and 6?

$|A| = 9$

(a) Choose a four element subset of A. There are "9 choose 4" ways to do this.

answer $\binom{9}{4} = C(9,4) = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$

(b) Choose 3 elements of $A - \{5\}$.

answer $\binom{8}{3} = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

(c) How many subsets do contain both 5 and 6?

answer $\binom{7}{2} = \frac{7 \cdot 6}{2} = 21$

answer to (c) $\binom{9}{4} - 21 = 126 - 21 = 105$

#6 (a) How many bit strings of length 10 are there?

(b) How many bit strings of length 10 contain five 1's?

Bit string is a string of 0's and 1's.
This is also what we call a word in $\{0,1\}$.

(a) _ _ _ _ _ _ _ _ _ _

2 choices 2 choices

$\#$ bit strings of length 10 $= \overbrace{2 \times 2 \times 2 \times \cdots \times 2}^{10 \text{ times}}$
$= 2^{10} = 1024$

(b) Bit strings of length 10 with exactly 5 1's.

_ _ _ _ _ _ _ _ _ _

Pick 5 out of slots in which to put 1's.
There $\binom{10}{5} = \frac{10!}{5!5!} =$

Bit strings of length 10 with at least 5 1's

$\binom{10}{5} + \binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} \stackrel{?}{=} 386$

$\binom{10}{5}$: exactly 5 1's; $\binom{10}{6}$: exactly 6 1's

$= |X_5| + |X_6| + \cdots + |X_{10}|$

$X_5 = \{$ bit strings of length 10 with exactly 5 ones $\}$

$X_6 = \{$ " " " " " 6 1's $\}$

X_7

X_8

X_9

X_{10}

$X_i \cap X_j = \emptyset$ when $i \neq j$

means sum principle applies