

Proposition For all sets A and B ,
 $A \cap B \subseteq B$.

idea: start with an element $x \in A \cap B$.

$x \in A \cap B$.

① work forward from this

Goal Show $x \in B$.

② work backward from this

provide argument for this.

idea Since $x \in A \cap B$ this means
 $x \in A$ and $x \in B$,

formal

proof: Let A and B be sets. Let x be an element of $A \cap B$.

This means that $x \in A$ and $x \in B$, by the definition of intersection.

This implies that $x \in B$. It follows that each element x in $A \cap B$ is also an element of B . Therefore $A \cap B$ is a subset of B by the definition of subset. \square