EXAM 1 **Name:**
Math 2513
10/2/19

Problem 1. *(10 points)* Let c and d be positive real numbers. Consider the implication statement: "*If* $c + d < 100$ *then* $c < 40$ *or* $d < 60$."
State the (a) converse, (b) contrapositive and (c) inverse of this implication in simplest form, and then (d) give a counterexample showing that at least one of these statements is false.

Problem 2. *(30 points)* Let A and B be sets. Give (a) an elementwise proof that

$$(A - B) \cup (B \cap A) \subseteq A$$

then (b) prove that the two sets $(A - B) \cup (B \cap A)$ and A are equal.

Problem 3. *(20 points)* Use a proof by contradiction to show that $(A - B) \cap (B - A) = \emptyset$ for all sets A and B.

Problem 4. *(10 points)* Let X be the set $X = \{\emptyset, \mathbb{Q}\}$ (where $\mathbb{Q}$ denotes the set of rational numbers) . Describe all of the subsets of X.

Problem 5. *(20 points)* Consider the following purported proof to the statement "*For all sets* A, B *and* C, $A - (B \cap C)$ *is a subset of* $(A - B) \cap (A - C)$".

Claimed Proof: Let A, B *and* C *be sets. Suppose that* x *is an element of* $A - (B \cap C)$. *This means that* $x \in A$ *and* $x \notin B \cap C$ *by the definition of set difference. Since elements of* $B \cap C$ *are in both sets* B *and* C *by the definition of intersection, it follows that* $x \notin B$ *and* $x \notin C$. *Since* $x \in A$ *and* $x \notin B$, *the definition of set difference implies that* $x \in A - B$. *Since* $x \in A$ *and* $x \notin C$, *the definition of set difference implies that* $x \in A - C$. *Therefore* x *is an element of both* $A - B$ *and* $A - C$, *which means that* $x \in (A - B) \cap (A - C)$ *by the definition of intersection. This shows that each element* x *in* $A - (B \cap C)$ *is also an element of* $(A - B) \cap (A - C)$, *and the proof is complete using the definition of subset.* □

(a) Clearly explain why this proof is incorrect.
(b) Give a counterexample that shows definitively that the statement is false.

Problem 6. *(15 points)* In the integer grid how many shortest paths p are there starting at $(0, 0)$ and ending at $(5, 3)$ such that:
(a) p passes through the point $(4, 1)$.
(b) p passes through the point $(1, 4)$.
(c) p does not contain any points of the form (n, n) except for $(0, 0)$.
Give some justification for your answers, and indicate some relevant paths and their associated strings of R's and U's on the attached grid sheet.