Discrete Mathematics
Some Permutation/Combination Problems — with brief answers

1. How many ways are there to select a group of 5 women from 16 husband/wife teams?
   answer: $4368 = C(16, 5)$

2. How many ways are there to arrange the 7 letters $AAABBBB$?
   answer: $35 = \binom{7}{3} = C(7, 3)$

3. How many ways are there to arrange the 12 letters of $AAABBBBCCCCC$ without having two C's together?
   answer: $1,960 = C(7, 4)C(8, 5)$. First arrange the A's and B's. Then choose 5 of the 8 interstices in which to place the C's.

4. How many ways are there to seat 10 people in a row?
   answer: $3,628,800 = 10! = P(10, 10)$

5. How many ways are there to seat 10 people at a round table?
   answer: $362,880 = 9!$. The point to be aware of here is that if the table is rotated then the relative positions of the people remains the same. So to set this problem up, first fix a seat where the first person is to sit. Then there will be 9 possibilities for the person sitting to his or her left, then 8 choices to fill in the seat to the left of the second person, and so on.

6. How many distinguishable dominoes are there? (Each of the two ends of domino has 0 to 6 dots on it.)
   answer: $28 = C(8, 2)$. Think of arranging 6 bars to separate 7 compartments corresponding to the 7 digits from 0 to 7, then arrange two stars to represent the two sides of the domino.

7. How many ways can 14 men and 9 women be seated in a row so that no 2 women sit next to each other?
   answer: $158,334,467,844,833,280,000 = 14! \cdot C(15, 9) \cdot 9!$. Arrange the men first then choose 9 of 15 interstices in which to sit the women, and finally choose an ordering of the 9 women.

8. How many ways are there to select 10 cans of soda from 4 different brands?
   answer: $286 = C(13, 3)$**. (3 bars and 10 stars.)**

9. How many ways can 22 cans of beer be handed out to 4 people if everyone must get at least one can?
   answer: $1,330 = C(21, 3)$. First give one can to each person. Then have 18 stars (for the remaining cans) and 3 bars (creating compartments for the 4 people).

10. How many ways are there to pick 9 cans from a tub containing 8 cans of 57 different varieties of soda?
    answer: $31,966,749,823 = C(65, 9) - 57$. (9 stars and 56 bars, but you can't choose all 9 cans to have the same brand.)

11. How many ways are there to distribute 5 apples and 8 oranges to 6 children?
    answer: $324,324 = C(10, 5)C(13, 8)$. First distribute the apples, then distribute the oranges.

12. How many ways are there to select some fruit from 5 apples and 8 oranges, taking at least one piece?
    answer: $53 = 6 \cdot 9 - 1$

13. How many non-negative integers less than a billion have five 7's?
    answer: $826,686 = C(9, 5)9^4$.
    An integer less than a billion (which equals 10^9) can be viewed as a string of length nine consisting of integers between 0 and 9. (Notice that in this problem it's OK if the integer starts off with some 0's since it is only required to have 9 or fewer digits.) First pick five of the nine positions in which to place the 7's then fill each of the remaining four positions with one of the nine possible integers between 0 and 9 that aren't equal to 7.

14. How many 5-letter words can be formed from the alphabet without repeating any letter?
answer: $7,893,600 = P(26,5) = 26!/(26-5)! = 26!/21!$

15. How many ways are there to pair off 8 women and 8 men at a dance?
answer: $40,320 = P(8,8) = 8!$. (There are 8 choices for a partner for the first woman, then 7 choices for the second, and so on.)

16. How many positive integer solutions are there to the equation $w + x + y + z = 24$?
answer: $1771 = C(23,3)$.
Each of $w-1$, $x-1$, $y-1$ and $z-1$ is non-negative and $(w-1)+(x-1)+(y-1)+(z-1) = 20$. Now count the number of strings of length 23 consisting of 3 bars and 20 stars (where the 3 bars create four compartments where the number of stars respectively give the values for $w-1$, $x-1$, $y-1$ and $z-1$.)

17. How many ways are there to pick 12 letters from 12 A's and 12 B's?
answer: $13 = C(13,1)$ which is the number of strings of length 13 with 1 bar and 12 stars (and the number of *'s to the left of the bar determines the number of A's to choose).

18. How many ways are there to pick 18 letters from 12 A's and 12 B's?
answer: $7 = C(7,1)$. At least 6 A's and 6 B's must be taken. This leaves 6 letters to choose from 6 A's and 6 B's.

19. How many ways are there to pick 25 letters from 12 A's, 12 B's and 12 C's?
answer: $78 = C(13,11)$. First pick 11 letters from the 12 A's, 12 B's and 12 C's. (The number of ways to do this equals the number of strings of length 13 consisting of 2 bars and 11 stars, where the bars separate off three compartments determining the number of A's, B's and C's respectively.) What is left over after choosing these 11 letters will be the desired 25 letters with no more than 12 of each letter.

20. How many ways are there to select a dozen doughnuts chosen from 7 varieties with the restriction that at least 1 doughnut of each variety is chosen?
answer: $462 = C(11,6)$. (5 stars and 6 bars)

21. How many ways are there to put 17 red balls into 12 distinguishable boxes with at least 1 ball in each box?
answer: $4,368 = C(16,11) = C(16,5)$