Discrete Math Group Project #2, 9/4/20

Instructions: Each team should submit one report for this assignment, due by Wednesday, 9/9. The report may be submitted either electronically by 6pm, or in written form at class. If you submit via email, please title your file as "Project2-Team*.pdf" (where * indicates your team number). Make sure you have a title at the top of your report which includes the names of all of your team members.

Intro: Take the first few minutes to introduce yourself and get acquainted with classmates in your team. Take time to discuss things like: why you are taking this course and how it fits into your academic plans; how long you've been at OU; where you're from; or etc.

Part I: Consider the following statement and its "proof".

CLAIM 1. *The integers* 1 *and* 2 *are equal.*

Proof. Let a and b be integers and assume that $a = b$. Multiplying both sides of the equation $a = b$ by a shows that $ab = a^2$, and adding a^2 to both sides gives

$$a^2 + ab = a^2 + a^2 = 2a^2.$$

By subtracting $2ab$ from both sides of this equation, we see that

$$2a^2 - 2ab = (a^2 + ab) - 2ab = a^2 - ab.$$

After factoring out 2, we can rewrite this as

$$2(a^2 - ab) = 1(a^2 - ab).$$

Now dividing both sides of this equation by $a^2 - ab$ shows that $2 = 1$. □

Obviously this statement is not true, so there must be a mistake in its purported "proof". Locate and describe the mistake, and explain why it is incorrect.

Part II:
Give answers for all of the problems in Part C (#29 through 38) of the "Exercises for Section 1.1" on page 8 of Hammack's book. Just listing answers will be OK for this problem.

Part III:
This part involves sg-paths in the square grid, as in group project 1. For each problem you should provide some written and/or pictorial evidence supporting your answer.

We say that an sg-path from the origin to a grid point (n, n) in the first quadrant where $n \in \mathbb{N}$ is a Catalan path provided that it never goes above the line $y = x$. (However it is allowed to include any number of grid points on the line $y = x$.)

(a) How many sg-paths are there from $(0, 0)$ to $(4, 4)$, and how many of these are Catalan paths?

(b) Give the R/U strings for all of the Catalan paths that you found in (a) which pass through the grid point $(2, 2)$.

(c) By examining the R/U string for an sg-path from $(0, 0)$ to (n, n) where $n \in \mathbb{N}$, how you can tell whether or not the path is Catalan? Illustrate your answer (giving both the R/U string and a picture of its grid path) for at least one example of an *sg*-path that is Catalan and one that is not Catalan.

(d) Explain why the number of Catalan paths from $(0, 0)$ to (n, n) is larger than the number of Catalan paths from $(0, 0)$ to $(n-1, n-1)$ where n is a natural number larger than 1.

(e) The number of Catalan paths from $(0, 0)$ to (n, n) is called a "Catalan number". Consult Wikipedia to determine some basic biographical information about Catalan.