FINAL EXAM **Name:**
Math 2513
12/15/20

Instructions: To receive credit you must provide explanations with each of your answers. In problems 1(a) and 2(b), a formal proof is required.

Problem 1. *(20 points)* For sets A, B and C let

$$X = A \cap (C - B) \quad \text{and} \quad Y = C - (A \cap B \cap C).$$

(a) Give an elementwise proof that X is a subset of Y.
(b) Show that Y is not necessarily a subset of X by giving a counterexample.

Problem 2. *(10 points)* Let a and b be integers. Consider the proposition: *If a and b are integers for which $5a^2b - 2b - 1$ is even then a is odd.*
(a) State the contrapositive of this proposition.
(b) Use (a) to prove that the proposition is true.

Problem 3. *(15 points)* Let $f : \mathbb{N} \to \mathbb{N}$ and $g : \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = (2n-5)^2$ and $g(n) = n^2$.
(a) Show that f is not injective. (b) Show that g is injective. (c) Show that f is not surjective.

Problem 4. *(15 points)* (a) How many bit strings of length 9 are there?
(b) How many bit strings of length 9 start and end with the string 001?

Problem 5. *(15 points)* Let X be the set consisting of the fourteen lower case letters from a to n. Find
(a) The number of 10 letter words that can be made using letters of X.
(b) The number of 10 letter words that can be made using letters of X where no letter is repeated.
(c) The number of unordered 10-element lists of X (with no repetition).
(d) The number of unordered 10-element lists of X where repetition is allowed.
(e) The number of unordered 10-element lists of X with repetition in which there is at least one a and one m.

Problem 6. *(10 points)* Let n be a natural number. Consider the set $\mathcal{S}_n$ of sg-paths in the integer grid from the origin to (n, n) such that each integer point (x, y) on the path satisfies the inequality $y \geq x/2$.
(a) List the R/U strings for all of the elements of $\mathcal{S}_1$.
(b) List the R/U strings for all of the elements of $\mathcal{S}_2$.
(c) How many elements does $\mathcal{S}_3$ have?
(d) Give some justification for the observation that $|\mathcal{S}_{n+1}|$ is bigger than $2|\mathcal{S}_n|$ for every $n \in \mathbb{N}$.

Problem 7. *(15 points)* Consider the set of 13 digit natural numbers in which the digits 1 and 9 each occur once, 3 and 7 each occur three times and 5 occurs five times.
(a) How many of these 13 digit numbers are there?
(b) In how many of these 13 digit numbers do all of the 1's, 3's and 5's occur before the 7's and the 9's?
(c) How many of these 13 digit numbers have the property that there are no two consecutive 5's.