EXAM 2 **Name:**
Math 2513
11/23/20

PROBLEM 1. *(20 points)* (a) Define what it means for a function $f : X \to Y$ to be surjective.
(b) Define what it means for a function $f : X \to Y$ to <u>not</u> be injective.

PROBLEM 2. *(20 points)* Give a formal proof showing that the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $f(m, n) = (m + n, m + 3n)$ is injective.

PROBLEM 3. *(10 points)* Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = n + (-1)^{n+1}$.
(a) What does $f(n)$ equal if n is even? if n is odd? Use your answer to express $f(n)$ with a piecewise formula depending on whether n is even/odd.
(b) Show that 2021 is in the range of f.
(c) Is the set $f([1, 2021])$ an interval in $\mathbb{N}$?
(Terminology: An "interval in $\mathbb{N}$" has the form $[a, b] = \{n \in \mathbb{N} | a \leq n \leq b\}$ for some $a, b \in \mathbb{N}$.)
(d) Is f surjective? Explain.

PROBLEM 4. *(20 points)* Consider $\mathbb{Z}_{15} = \{0, 1, \ldots, 14\}$.
(a) 7 has a additive inverse in $\mathbb{Z}_{15}$. What does it equal?
(b) 7 has a multiplicative inverse in $\mathbb{Z}_{15}$. What does it equal?
(c) Use your answer to (b) to solve the linear equation $7x + 4 = 0$ in $\mathbb{Z}_{15}$.
(d) Show that the linear equation $7x + b = 0$ has a solution in Z_{15} for any $b \in \mathbb{Z}_{15}$, and find it.

PROBLEM 5. *(20 points)* Consider the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
(a) How many subsets with four elements does A have? Give your answer both as an integer and using the "choose" notation.
(b) How many subsets of A have four elements and contain 5?
(c) How many subsets of A have four elements but do not contain both 5 and 6?

PROBLEM 6. *(10 points)* Let W be the set of bit strings with length 10.
(a) How many elements does W have? Briefly justify your answer.
(b) How many elements in W contain 5 ones? Briefly justify your answer.
(c) Define a function $F : W \to \mathbb{Z}_{\geq 0}$ by assigning to each string in W the number of substrings of the form 000 (that is, three consecutive 0's in the string). For example, $F(0100001000) = 3$. What is the range of F?
(d) How many elements $w \in W$ have $F(w) = 4$?