

EXAM 3
Math 2513
7-15-05

Name _____

1. (15 points) Use the Principle of Mathematical Induction to prove that $n!$ is greater than 3^n whenever n is an integer larger than 6.
2. (10 points) a) Write out rows 0 through 6 of Pascal's triangle (using the convention that the first row in Pascal's triangle is row 0.) b) Which row of Pascal's triangle has the number 165 in the fourth position from the left? c) Does the number 165 appear in any row other than the one you found in (b)? If so, what are the rows?
3. (15 points) Two sets A and B contain 4 and 5 elements respectively (that is, $|A| = 4$ and $|B| = 5$). (a) With only this information available about the sets A and B , what are the possible values for $|A \cup B|$? (b) If $|A \cup B| = 9$ what can you conclude about the sets A and B ? (c) If $|A \cup B| = 5$ what can you conclude about the sets A and B ?
4. (15 points) Let \mathcal{B}_8 denote the set which consists of all bit strings of length 8.
 - (a) How many elements does \mathcal{B}_8 have?
 - (b) How many elements of \mathcal{B}_8 start with two 1's and end with two 0's?
 - (c) How many elements of \mathcal{B}_8 contain exactly four 1's?
 - (d) How many elements of \mathcal{B}_8 either contain exactly four 1's, or start with two 1's and end with two 0's?
5. (15 points) As in the previous problem, let \mathcal{B}_8 denote the set which consists of all bit strings of length 8.
 - (a) How many elements of \mathcal{B}_8 contain a string of five 1's?
 - (b) How many subsets does \mathcal{B}_8 have?
 - (c) How many subsets A does \mathcal{B}_8 have in which every element of A contains a string of five 1's?
 - (d) How many subsets A does \mathcal{B}_8 have in which at least one element of A has a string of five 1's?
6. (15 points)
 - (a) How many 11-letter words can be formed using the letters of *CONNECTICUT*?
 - (b) How many 11-letter words can be formed using the letters of *CONNECTICUT* if all three *C*'s are consecutive?
 - (c) How many 11-letter words can be formed using the letters of *CONNECTICUT* if no two *C*'s are consecutive?
7. (10 points) Let $X = \{A, B, C, D, E, F, G\}$. Work out the actual numbers for each of the following. (a) Determine the number of 4-permutations of X . (b) Determine the number of 4-combinations of X . (c) Determine the number of 4-permutations of X where repetition is allowed. (d) Determine the number of 4-combinations of X where repetition is allowed.
8. (15 points) How many ways are there to roll three dice?