EXAM 2 Name ____________________
Math 2513
7-1-05

1. *(10 points)* Show that the composition of two one-to-one functions is one-to-one.

2. *(10 points)* For each of the following statements, write an English sentence which describes the negation of that statement in the most direct way.
   a) This week it will rain on Saturday.
   b) This week it will rain on Saturday and on Sunday.
   c) If it rains on Saturday this week then it will not rain on Sunday.

3. *(15 points)* Let a, b and c be positive integers. Show that if a divides b and b divides c then a divides c.

4. *(15 points)* Demonstrate how the Euclidean algorithm works by finding the greatest common divisor of 5720 and 12342.

5. *(15 points)* Use either an indirect proof or a proof by contradiction to show that if k is an integer and $5k + 4$ is odd then k is odd. Before starting on your proof write a short paragraph stating which technique you will use and outlining what you will need to show to carry out the proof.

6. *(10 points)* Consider the statement of the previous problem: *if k is an integer and $5k + 4$ is odd then k is odd.* Write sentences stating (a) the converse of this statement, and (b) the contrapositive of this statement.

7. *(15 points)* Use Mathematical Induction to prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1$. (Write this out carefully you will graded on both correctness and organization.)

8. *(10 points)* Let $A = \{1, 2, 3\}$.
   a) There are 8 different relations on A which are reflexive, contain $(1, 2)$ and do not contain $(1, 3)$. List all 8 of these relations.
   b) Of the 8 relations in part (a) make a chart indicating which are symmetric, which are anti-symmetric and which are transitive. (You don't need to justify your answers.)
   c) Determine the matrix M associated to one of the 8 relations which is not symmetric and explain how you can see from M that the relation is not symmetric.
   d) Draw the directed graph Γ associated to one of the 8 relations which is not transitive and explain how you can see from Γ that the relation is not transitive.