

Class Problem
Math 2513
Monday, July 18

PROBLEM.

- (a) State the binomial theorem.
- (b) Use (a) to expand $(A + B)^5$ as a degree five polynomial in A and B .
- (c) Use (a) to expand $(2A - B)^5$ as a degree five polynomial in A and B .

ANSWERS:

- (a) For real variables x and y and for $n \in \mathbb{N}$,

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k.$$

- (b)

$$\begin{aligned} (A + B)^5 &= \binom{5}{0} A^5 + \binom{5}{1} A^4 B + \binom{5}{2} A^3 B^2 + \binom{5}{3} A^2 B^3 + \binom{5}{4} A B^4 + \binom{5}{5} B^5 \\ &= A^5 + 5A^4 B + 10A^3 B^2 + 10A^2 B^3 + 5A B^4 + B^5 \end{aligned}$$

- (c)

$$\begin{aligned} (2A - B)^5 &= (2A)^5 + 5(2A)^4(-B) + 10(2A)^3(-B)^2 + 10(2A)^2(-B)^3 + 5(2A)(-B)^4 + (-B)^5 \\ &= 32A^5 - 80A^4 B + 80A^3 B^2 - 40A^2 B^3 + 10A B^4 - B^5 \end{aligned}$$