Class Problem
Math 2513
Norman, July 14

PROBLEM.
(a) How many "words" of length 14 can be made using the letters of $NORMANOKLAHOMA$?
(b) Determine the number of 14-letter words formed from $NORMANOKLAHOMA$ contain three consecutive O's.
(c) How many "words" of length 14 made from $NORMANOKLAHOMA$ have no consecutive O's?

SOLUTIONS:

(a) By the Multinomials Theorem, the number of permutation of $\{N, O, R, M, A, K, L, H\}$ with three O's and A's, two N's and M's and one R, K, L and H is

$$\binom{14}{3,3,2,2,1,1,1,1} = \frac{14!}{3!3!2!2!} = 605,404,800.$$

NOTE: The numerical value is only included to give a relative idea of the size of the number.
(b) The number of permuations of $\{N, OOO, R, M, A, K, L, H\}$ with three A's, two N's and M's and one OOO, R, K, L and H is

$$\binom{12}{3,2,2,1,1,1,1,1} = \frac{12!}{3!2!2!} = 19,958,400.$$

(c) We break the task of listing words of the indicated type with no consecutive O's into two subtasks: First choose an 11-letter word with three A's, two N's and M's and one R, K, L and H. Second, choose 3 of the 12 gaps in the word resulting from the first task and insert one O into each of these 3 gaps. There are $\binom{11}{3,2,2,1,1,1,1}$ to perform the first task and $\binom{12}{3}$ ways to perform the second. So the answer to (c) is

$$\binom{11}{3,2,2,1,1,1,1}\binom{12}{3} = \frac{11!}{3!2!2!}\frac{12!}{9!3!} = 365,904,000.$$