

Class Problem
Math 2513
Wednesday, July 13

PROBLEM. (a) How many bit strings of length 7 contain a substring of the form 100?
(b) How many bit strings of length 7 contain a substring of the form 101?

COMMENT: There are only $2^7 = 128$ bit strings of length 7, so one way to approach these problems would be to list all 128 of these and then cross out those that don't satisfy (a) or (b). Please don't solve the problem this way.

ANSWERS:

(a) The number of bit strings of length 7 that contain a substring of the form 100 is 74: Let A be the set of all such bit strings of length 7. Then

$$A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$$

where A_i is the set of all bit strings of length 7 which have the first occurrence of 100 starting in position i . Then $|A_1| = 2^4$, $|A_2| = 2^4$, $|A_3| = 2^4$, $|A_4| = 2^4 - 2$ and $|A_5| = 2^4 - 4$. Since the five subsets are pairwise disjoint, the sum principle gives that

$$|A| = 2^4 + 2^4 + 2^4 + (2^4 - 2) + (2^4 - 4) = 74.$$

(b) The number of bit strings of length 7 that contain a substring of the form 101 is 63. Here we can reduce the problem to computing the sums of the cardinalities of five subsets similar to (a), and the respective orders of these subsets are 2^4 , 2^4 , $2^4 - 4$, $2^4 - 6$ and $2^4 - 7$.