Class Problem
Math 2513
June 8, 2005

PROBLEM. Consider the statement: *"For all sets A and B, $A \cap B \subseteq A$."*
(1) Identify each of the basic set theory concepts that are involved in this statement, and write out the definitions of each of these concepts.
(2) Prove the statement.

Solution:

(1) The two set theory concepts that are involved in this statement are 'intersection' and 'subset'.

- If A and B are sets then an object x is an element of the *intersection* $A \cap B$ if and only if $x \in A$ and $x \in B$.

- If A and B are sets then we say that B *is a subset of* A provided that if x is an element of B then x is an element of A.

COMMENT: These definitions are complete and very precise, so you will be able to base your proof entirely on these descriptions. In fact an acceptable proof will rely entirely on these descriptions. While you certainly have previous experience and understanding of the basic concepts of intersection and subset (which should help you greatly in seeing the correct to approach a problem like this), you should block that out when you construct your proof and strictly base the proof on the precise definitions that we have given. For example, it is important to avoid any language in your proof that suggests thinking in terms of Venn diagrams.

(2) Here's a proof that $A \cap B$ *is a subset of A for all sets A and B*:

Proof. Let A and B be sets. Suppose that x is an element of $A \cap B$. By the definition of intersection, this means that $x \in A$ and $x \in B$. Therefore x is an element of A. So we have shown that every element of $A \cap B$ is also an element of A. Using the definition of subset this means that $A \cap B$ is a subset of A, which completes the proof. $\square$