

**Class Problem**  
**Math 2513**  
**Tuesday, June 28**

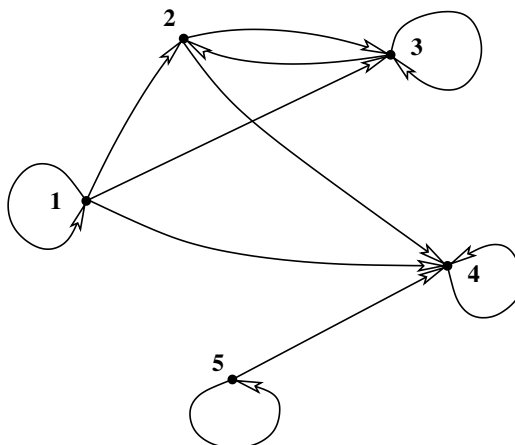
PROBLEM. Let  $A = \{1, 2, 3, 4, 5\}$ . The directed graph shown below determines a relation  $R$  on  $A$  using the convention that if there is a directed edge from  $x$  to  $y$  then  $(x, y)$  is an element of  $R$ .

(a) What is  $R$  for this example?

(b) Which of the following properties does this relation satisfy:

reflexive, symmetric, anti-symmetric, transitive ?

Justify each of your four answers with an explanation or a counterexample as appropriate.



ANSWERS:

(a) The relation on  $A$  determined by the directed graph is

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 2), (3, 3), (4, 4), (5, 4), (5, 5)\}.$$

(b) This relation does not satisfy any of the four properties.

1.  $R$  is not reflexive because  $(2, 2) \notin R$ .
2.  $R$  is not symmetric because  $(1, 2) \in R$  but  $(2, 1) \notin R$ .
3.  $R$  is not anti-symmetric because  $(2, 3) \in R$  and  $(3, 2) \in R$  but  $2 \neq 3$ .
4.  $R$  is not transitive because  $(3, 2) \in R$  and  $(2, 4) \in R$  but  $(3, 4) \notin R$ .

NOTE: See if you can interpret the answers to part (b) in terms of the directed graph and in terms

of the matrix for  $R$ , which is the  $5 \times 5$  matrix  $M = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ .