Class Problem
Math 2513
Friday, June 24

Please feel free to collaborate and/or compare notes with your classmates on this problem.

PROBLEM. (a) Let N be the positive integer $N = 25!$. Find all of the prime numbers which divide N, then work out the complete prime factorization of N.
(b) In part (a) you should have determined that the 7-part of 25! equals 7^3 (see definition below). Are there any other positive integers k besides $k = 25$ for which 7^3 is the 7-part of $k!$? If so, find them.

Definition. Let a be a positive integer whose prime factorization is

$$a = p_1^{n_1} p_2^{n_2} p_3^{n_3} \cdots p_r^{n_r}$$

where p_1, p_2, $\ldots$, p_r are distinct primes and each n_i is a natural number. Then the *p_i-part of a* is defined to be $p_i^{n_i}$ for $i = 1, \ldots, r$.

ANSWERS:

(a) The prime divisors of $N = 25!$ are all of the prime numbers less than or equal to 25. These are 2, 3, 5, 7, 11, 13, 17, 19 and 23. The prime factorization of N is

$$N = 2^{22} 3^{10} 5^6 7^3 11^2 13^1 17^1 19^1 23^1.$$

Incidentally, 25! equals 15511210043330985984000000.

(b) If k equals 21, 22, 23, 24, 25, 26 or 27 then the 7-part of $k!$ is 7^3. If k is larger than 27 then the 7-part of $k!$ is larger than 7^3. (For example, the 7-part of 28! is 7^4.) If k is smaller than 21 then the 7-part of $k!$ is less than 7^3. (For example, the 7-part of 20! is 7^2.)