Class Problem
Math 2513
Wednesday, June 15

Problem. Use a proof by contradiction and basic definitions to prove that $(A - C) \cap (C - B) = \emptyset$ for all sets A, B and C.

Proof. Let A, B and C be sets. Assume that $(A - C) \cap (C - B)$ is not the empty set. Then $(A-C)\cap(C-B)$ contains at least one element. Let x be such an element, that is $x \in (A-C)\cap(C-B)$. By the definition of intersection this means that (1) $x \in A-C$ and (2) $x \in C-B$. Using the definition of set difference, it follows from (1) that $x \in A$ and $x \notin C$, and so, in particular, x is not an element of C. From (2) and the definition of set difference, it follows that $x \in C$ and $x \notin B$, and from this we deduce that x is an element of C. Therefore it has been shown that x is both an element of C and not an element of C, which is a contradiction. We conclude that the assumption that $(A - C) \cap (C - B)$ is not the empty set must be false, and this proves the result. $\square$