EXAM 1 Name ______________________
Math 2513
9-17-04

1. *(15 points)* Consider the set identity $A - B \subseteq A$.
(a) Write down complete statements of the two definitions which play central roles in this identity.
(b) Prove the identity.

2. *(10 points)* In one or two sentences describe the standard general strategy which you would use to prove that two given sets are equal.

3. *(15 points)* Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function described by the rule $f(n) = n^2 + 1$. (a) What are the domain, codomain and range of this function f? (b) Show (by example) that f is not injective. (c) Determine $f(S)$ if $S = \{-1, 0, 1, 2, 3\}$.

4. *(15 points)* Let A and B be sets such that $A \subseteq B$. Show that $A \cup B \subseteq B$.

5. *(10 points)* Let A and B be two non-empty sets. (a) Describe the Cartesian product $A \times B$. (b) Let $P : A \times B \to B$ be the function which maps (a, b) to b. Show that P is onto.

6. *(10 points)* Let f and g be functions from $\mathbb{R}$ to $\mathbb{R}$ given by $f(x) = 3x - 2$ and $g(x) = ax + b$ where a and b are constants. (a) Determine an equation for $g \circ f(x)$ (b) Find values for a and b such that $g \circ f(x) = x$ for every $x \in \mathbb{R}$. Then determine $f \circ g(x)$ when these values of a and b are used.

7. *(10 points)* Explain why the function $h : \{a, b, c, d\} \to \{1, 2, 3, 4\}$ defined by $h(a) = 2$, $h(b) = 4$, $h(c) = 3$, and $h(d) = 1$ is a bijection.

8. *(15 points)* Let A, B and C be sets. Prove that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.