Math 2433  
Instructor: Ajit Bhand  
Quiz 4  
Name: __________________

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Instructions

• Explain your answers clearly. Credit will be given only if you provide sufficient justification for your answers.

• Non-graphing calculators are permitted.

1. [4 points] Draw and describe the graph of $y = 3x - 4$ in $\mathbb{R}^3$.

Solution: The given equation represents a line in $\mathbb{R}^2$. Since the $z$ variable is not present, it can take all real values. The graph in $\mathbb{R}^3$ is therefore a plane parallel to the $z$ axis which intersects the $x - y$ plane in the line $y = 3x - 4$. 
2. [6 points] (a) Show that the equation \( x^2 + y^2 + z^2 = 4x - 2y \) represents a sphere, and find its radius and center.

(b) What does the region \( x^2 - 4x + y^2 + 2y + z^2 < 9 \) represent in \( \mathbb{R}^3 \)?

**Solution:**

(a) The equation is \( x^2 + y^2 + z^2 - 4x + 2y = 0 \). Completing the squares, we get

\[
(x - 2)^2 + (y + 1)^2 + z^2 = 5
\]

which represents a sphere centered at \((2, -1, 0)\) with radius \( \sqrt{5} \).

(b) Completing the squares, the inequality reads

\[
(x - 2)^2 + (y + 1)^2 + z^2 < 9 + 5 = 14
\]

which represents the set of points strictly inside the sphere of radius \( \sqrt{14} \) and center \((2, -1, 0)\).