Last names and Professions

Messrs. Baker, Dyer, Farmer, Glover and Hosier are seated around a circular table, playing cards. Each person has the namesake of the profession of one of the others. The dyer is seated two places to the left of Mr. Hosier. The baker sits two places to Mr. Baker’s right. The farmer is seated to the left of Mr. Farmer. Mr. Dyer is on the glover’s right.

1. What is the name of the dyer?

2. What can you say about the profession of Mr. Baker?

**Answer.** Draw a picture of a pentagonal table from above. The 4 pictures below encode the 4 statements about the relative positions of people.

![Diagram](image)

From these diagrams it is easy to read off illegal configurations of people (read in a counterclockwise direction):

\[ BH, \quad HF, \quad HD, \quad B * F, \quad B * D \]

where \( * \) denotes a single space at the table. These are not allowed since they will force two people with different professions to occupy the same place. For example, \( BH \) is not allowed since it would force \( d \) and \( b \) to occupy the same place (two slots to the left of \( H \)).

Similarly, there is a list of illegal configurations arising from the fact that a person occupying a particular slot can not have the namesake of his profession.

\[ H * D, \quad DG \]

Now generate possible table seating configurations. If we start by looking two places to the left of \( B \) we realize that it can’t be occupied by \( B \) (clearly!) or by \( D \) or \( F \); the latter two are ruled out by our first list of impossible configurations. This gives the following two partial configurations.

![Partial configurations](image)

Starting from each of these partial configurations, fill in other places with people, keeping in mind the lists of illegal configurations above, and the fact that the more people that you seat in a configuration the fewer names that you will have left to put in the remaining free slots. In fact, there is precisely one way to fill in these two configurations, as the following pictures indicate. It is fun to supply reasons for each of the arrowed steps below.
These are the only two possible configurations of the 5 people about the table which satisfy all the conditions above. In both configurations we see that the dyer must be Mr. Farmer, so we have an answer to the first question. However, we also see that Mr. Baker is either a hosier or a farmer, but that there is not enough information in the question to narrow down his occupation any further.