## Main results

**Real Analysis I and II, MATH 5453-5463, 2006-2007**

<table>
<thead>
<tr>
<th>Section</th>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction.</strong></td>
<td></td>
</tr>
<tr>
<td>1.3 Operations with sets. DeMorgan Laws.</td>
<td></td>
</tr>
<tr>
<td>1.4 Proposition 1. Existence of the smallest algebra containing C.</td>
<td></td>
</tr>
<tr>
<td>2.5 Open and closed sets.</td>
<td></td>
</tr>
<tr>
<td>2.6 Continuous functions. Proposition 18.</td>
<td>Hw #1. p.16 #9, 11, 17, 18; p.19 #19. p.49 #40, 42, 43; p.53 #53*.</td>
</tr>
<tr>
<td>2.7 Borel sets.</td>
<td></td>
</tr>
<tr>
<td><strong>3.2 Outer measure.</strong></td>
<td></td>
</tr>
<tr>
<td>Proposition 1. Outer measure of an interval.</td>
<td></td>
</tr>
<tr>
<td>Proposition 2. Subadditivity of the outer measure.</td>
<td></td>
</tr>
<tr>
<td>Proposition 5. Approximation by open sets.</td>
<td></td>
</tr>
<tr>
<td><strong>3.3 Measurable sets.</strong></td>
<td>Hw #2. p.55 #1-4; p.58 # 7, 8.</td>
</tr>
<tr>
<td>Lemma 6. Measurability of sets of outer measure zero.</td>
<td></td>
</tr>
<tr>
<td>Lemma 7. Measurability of the union.</td>
<td></td>
</tr>
<tr>
<td>Theorem 10. Measurable sets form a sigma-algebra.</td>
<td></td>
</tr>
<tr>
<td>Lemma 11. Interval is measurable.</td>
<td></td>
</tr>
<tr>
<td>Theorem 12. Borel sets are measurable.</td>
<td></td>
</tr>
<tr>
<td>Proposition 13. Sigma additivity of the measure.</td>
<td></td>
</tr>
<tr>
<td>Proposition 14. Continuity of the measure.</td>
<td></td>
</tr>
<tr>
<td>Proposition 15. Approximation by open and closed sets.</td>
<td>Hw #3. p.64 #9-11, 13, 14.</td>
</tr>
<tr>
<td><strong>3.4 A nonmeasurable set.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>3.5 Measurable functions.</strong></td>
<td>Hw #4. p.70 #18-22.</td>
</tr>
<tr>
<td>Proposition 18. Equivalent definitions of measurability.</td>
<td></td>
</tr>
<tr>
<td>Proposition 19. Sums and products of measurable functions.</td>
<td></td>
</tr>
<tr>
<td>Theorem 20. Infima and suprema of measurable functions.</td>
<td></td>
</tr>
<tr>
<td><strong>3.6 Littlewood's three principles.</strong></td>
<td></td>
</tr>
<tr>
<td>Egoroff's theorem.</td>
<td></td>
</tr>
<tr>
<td>Lusin's theorem.</td>
<td></td>
</tr>
<tr>
<td><strong>4.2 Prop.2. Lebesgue's integral of a simple function and its props.</strong></td>
<td></td>
</tr>
<tr>
<td>Lebesgue's integral of a bounded measurable function.</td>
<td></td>
</tr>
<tr>
<td>Proposition 3. Criterion of integrability.</td>
<td></td>
</tr>
<tr>
<td>Proposition 5. Properties of integrals of bounded functions.</td>
<td></td>
</tr>
<tr>
<td><strong>4.3 Lebesgue integral of a nonnegative function and its properties.</strong></td>
<td>Hw #5. p.89 #3-4, 6-7, 9.</td>
</tr>
<tr>
<td>Theorem 10. Monotone Convergence Theorem.</td>
<td></td>
</tr>
<tr>
<td>Proposition 14. Continuity of the integral.</td>
<td></td>
</tr>
<tr>
<td><strong>4.4 Lebesgue integral of a general function.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Proposition 15. Properties of integrals of general functions.  
Theorem 16. Lebesgue Dominated Convergence Theorem.

Hw #6. p.93 #10(a), 11-15.

5.1 Derivates of a function.  
Vitali Lemma (no proof).  
Theorem 3. Differentiation of a monotone function.  

Hw #7. p.101 #1-5.

5.2 Functions of bounded variation.  
Lemma 4. Properties of the variations.  
Theorem 5. A BV function is the difference of increasing funcs.  

Hw #8. p.104 #7-11.

5.3 Differentiation of an integral.  
Lemma 7. Indefinite integral is continuous and BV.  
Lemma 8. Indefinite integral is 0 implies f=0.  
Theorem 10. (includes Lemma 9) G'(x)=f(x) a.e.

5.4 Absolutely continuous functions.  
Lemma 11. AC function is BV.  
Lemma 13. If f is AC, and f'(x)=0 a.e, then f=const.  
Theorem 14. f is an indefinite integral if and only if it is AC.  

Hw #9. p.110 #12, 14, 15, 18, 20(a,b).

5.5 Convex functions.  
Propositions 18-19. f is convex iff f''>=0.  
Proposition 20. Jensen Inequality.  

Hw #10. p.116 #23(a,b), 25-28.

6.1 Lp spaces.  
6.2 Theorem 1. Minkowski Inequality.  
Theorem 4. Holder Inequality.  

Hw #11. p.119 #1-4; p.122 #7.

6.3 Convergence and completeness.  
Proposition 5. Criterion of completeness.  
Theorem 6. Riesz-Fischer Theorem.  


6.4 Approximation in Lp.  
Lemma 7. Approximation by bounded functions.  
Proposition 8. Approximation by step and continuous functions.

S.1 Hilbert spaces.  
Cauchy-Schwarz inequality.  
Theorem. Closed unit ball in H is not compact.  
Theorem. Linear functional is continuous iff it is bounded.  
Projection Theorem.  
Riesz Representation Theorem for Hilbert spaces.

6.5 Proposition 11. g in Lq defines a bounded functional on Lp.  
Theorem 13. Riesz Representation Theorem (no proof).

Uniform limit of continuous functions.
Weierstrass M-test for continuous functions.
First Dini's Theorem.
Urysohn's lemma.
Tietze's Extension Theorem.  
Hw #1. Chap. 7 #1, 4, 7-8, 11(a), 12-16.

9.9 Second Dini's Theorem.
Kakutani-Krein Theorem.
Stone-Weierstrass theorem  
Hw #2. p. 213 #42-46; S. 1.1, 1.2.

11.1 Nonnegative measure.

Hw #3. Chap. 11 #1, 3-5, 7, 10-13.

11.3 Integration.
Theorem 11. Fatou's lemma.
Theorem 16. Lebesgue's Dominated Convergence Theorem.  
Hw #4. p.267 #17, 19-22.

11.5 Signed measures.
Propositions 19-20. Positive sets.
Hw #5. p. 275 #27(a), 29-31.

11.6 Lemma 9. Measurable function on nested sets.
Theorem 23. Radon-Nikodym Theorem.
Hw #6. p. 279 #33(a)-37.

11.7 Lemma. g in Lq defines a continuous linear functional on Lp.
Lemma 27. Criterion for g to be in Lq.

12.1 Outer measure.
Theorem 1. Outer measure generates a measure (no proof).  
Hw #7. S. 2.1-2.3; p. 291 # 1, 2.

12.2 Semialgebras.
Theorem C.1. Extension from a semialgebra.

12.4 Product measures.
Lemma C.3. Measurable rectangles form a semialgebra.
Theorem C.4. Set function on a semialgebra.
Examples of functions measurable on product spaces.
Lemma 15. Measurability of cross-sections.
Lemma 17. Sets of measure zero.
Proposition 18. Area by iterated integrals.
Theorem 20. Tonelli's Theorem.
Theorems 19. Fubini's Theorem.
Existence of the convolution f*g for integrable f and g.  
Hw #8. p. 310 #19, 21, 22, 30; S. 3.1-3.3.
10.1 Banach spaces.
10.2 Dual spaces.
   Proposition 10.3. $X^*$ is a Banach space.
   Theorem. $C(K)$ is a Banach space.
7.10 Equicontinuous sets in $C(K)$.
   Integral operators in $C[0,1]$.
   Theorem. Baire and Borel sets in metric spaces.
   Theorem. Regularity of Borel measures on compact metric spaces.
13.4-5 Theorem. Decomposition of functionals into $F^+$ and $F^-$. 
   Theorem. Representation of positive functionals.
   Riesz-Markov Representation Theorem for $[C(X)]^*$.
   Hw #9. p. 218 #1-3; p. 222 #13-14; p. 169 #47, 50.

S.1 Theorem S.1.1. Approximation by simple functions.
   Theorem S.1.2. Approximation by continuous functions.
   Mollifiers.
   Theorem S.2.2. Differentiation of an integral w/r to a parameter.
   Fourier Transform of a Gaussian.
   Theorem S.2.3. Regularization of functions.
   Theorem S.2.4. Approximation by smooth functions.
   Hw #10. S. 4.1-4.5.
   Hw #11. S. 5.2-5.6.