Be sure that this examination has 2 pages.

The University of Oklahoma

Qualifying Examination Examinations - August 14, 2019

Real Analysis

Dr. M. Zhu

Close book examination | Time: 180 minutes

Choose 7 out of the following 8 questions

Marks

[20] **1.** Prove: Any open subset O in R can be written uniquely as a countable union of disjoint open intervals.

[20] **2.** State and prove Egorov Theorem concerning a sequence of measurable functions.

[20] **3.** (1). State and prove the Fator Lemma.
(2). Find a sequence of nonnegative functions $\{f_n\}$ such that $\lim_{n\to\infty} f_n(x) = f(x)$ $a.e$, and

$$\int f < \liminf_{n\to\infty} \int f_n.$$

[20] **4.** Assume that $f(x)$ is continuous in $[0, \infty)$ and $\lim_{x\to\infty} f(x) = A$. Prove that for any $L > 0$

$$\lim_{n\to\infty} \int_0^L f(nx)dx = LA.$$

[20] **5.** Suppose f is integrable on R^n. For any $\epsilon > 0$, prove
(1) There is a ball B, such that

$$\int_{B^c} |f| < \epsilon.$$

(2) There is a $\delta = \delta(\epsilon) > 0$ such that

$$\int_E |f| < \epsilon, \quad whenever \quad m(E) < \delta.$$

[20] **6.** Assume that $f(x),\ g(x) > 0$ on a measurable set $E \subset R$. Assume that $p \in (0, 1)$ and $q = p/(p-1) < 0$. Prove that

$$\int_E fg dx \geq (\int_E f^p dx)^{1/p} \cdot (\int_E g^q dx)^{1/q}.$$

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[20] **7.** Prove that $L^p(E)$ (for $p \geq 1$) is complete.

[20] **8.** Let $\{e_i\}_{i=1}^{\infty}$ be an orthonormal basis for $L^2[0,1]$. Show that $e_i \rightharpoonup 0$. Could e_i strongly converge to a function $f \in L^2[0,1]$ (that is: $||e_i - f||_{L^2[0,1]} \to 0$)?

[160] **Total Marks**

The End