1. State the Fundamental Theorem of Algebra.

2. Find the eigenvalues and the eigenvectors of the following matrix

\[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}.
\]

3. Let \( V \) be a finite dimensional vector space over a field \( F \) and let \( T : V \to V \) be a linear map such that \( \text{Ker}(T) \cap \text{Im}(T) = \{0\} \). Show that

\[
V = \text{Ker}(T) \oplus \text{Im}(T).
\]

4. State Spectral Theorem for the finite dimensional vector spaces, with an inner product, over \( \mathbb{R} \) or \( \mathbb{C} \).

5. Let \( V \) be an \( n \)-dimensional vector space over \( \mathbb{R} \), with an inner product, and let \( A \subseteq \text{End}(V) \) be a vector subspace of mutually commuting self adjoint linear maps \( L : V \to V \). What is the maximal dimension of \( A \)?

6. State the Structure Theorem for the finitely generated abelian groups.

7. How many abelian groups, up to an isomorphism, of order 27 are there?

8. Let \( G \) be a group in which all elements other than the identity have order 2. Prove that \( G \) is abelian.

10. State Sylow Theorem.

11. How many elements of order 7 are there in a simple group of order 168?
12. Decompose $\mathbb{R}^2$ into the disjoint union of orbits under the action of the orthogonal group $O_2$.

13. Let $\phi : \mathbb{R} \to \mathbb{R}$ be a ring homomorphism such that $\phi(1) = 1$. Show that $\phi$ is the identity map. (Here $\mathbb{R}$ is the field of real numbers.)

14. Suppose $R$ is an integral domain and $F \subseteq R$ is a subring that is a field. Prove that if the dimension of $R$, viewed as a vector space over $F$, is finite that $R$ is also a field.

15. What is a necessary and sufficient condition on a positive integer $N$ so that the positive square root $\sqrt{N} \in \mathbb{Q}(2^{1/3})$?

16. Give an example of two different algebraically closed fields $E$ and $F$, such that $E$ is a subfield of $F$.

17. Give an example of a unique factorization domain which is not a principal ideal domain.

18. Give an example of a ring $R$ and a prime ideal $I \subseteq R$ which is not maximal.