Friday 02/14/2014 | Midterm I | 50 minutes

Name: ____________________ Student ID: ____________________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.
4. No calculators, no notes, no books.

Question	Points	Your Score
Q1	25	
Q2	25	
Q3	13	
Q4	13	
Q5	12	
Q6	12	
TOTAL	100	

Q1]... [25 points] Suppose f is a function whose domain is all the real numbers. Consider the following expression involving inputs x and a.

$$\frac{f(x) - f(a)}{x - a}$$

What is the expression above called?

Write down two interpretations of this expression.

Suppose that the following limit exists.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

What is this limit called?

Write down two interpretations of this limit.

Q2]... [25 points] Compute the following limit. Show all the steps of your work.

$$\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$$

Use the value of the limit above to write down the equation of the tangent line to the graph $y = x^2$ at the point $(3, 9)$.

Q3]. . . [13 points] Suppose that $\cos(t) = 2/\sqrt{5}$ and that $-\pi/2 \leq t \leq 0$. Draw a picture showing the angle t, and then determine the values of $\sin(t)$ and $\cot(t)$.

Q4]. . . [13 points] Let f be the piecewise defined function

$$f(x) = \begin{cases} 2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$$

Is f continuous at 0? Give reasons for your answer.

Q5]... [12 points] Suppose that $f(x)$ is a function whose domain is all real numbers, and suppose that

$$\lim_{x \to 1} \frac{f(x) - 8}{x - 1} = 2014$$

Use the information above to find the following limit

$$\lim_{x \to 1} f(x)$$

Show the steps of your work.

Q6]... [12 points] Show that the equation

$$x^5 - x^3 + 3x - 5 = 0$$

has a solution in the interval $(1, 2)$. Give the name of any theorem from class notes that you use.

Q1]... [25 points] Suppose f is a function whose domain is all the real numbers. Consider the following expression involving inputs x and a.

$$\frac{f(x) - f(a)}{x - a}$$

What is the expression above called?

THE Difference Quotient of f(x)

Write down two interpretations of this expression.

① It is the slope of the secant Line between the points (x, f(x)) and (a, f(a))

② It is the average rate of change of f(x) with respect to x over the interval [x, a].

Suppose that the following limit exists.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

What is this limit called?

The Derivative of f(x) at a (denoted by f '(a)).

Write down two interpretations of this limit.

① It is the slope of the tangent Line to the Graph y = f(x) at the point (a, f(a)).

② It is the instantaneous rate of change of f(x) with respect to x at the input a.

Q2]...[25 points] Compute the following limit. Show all the steps of your work.

$$\lim_{h\to 0} \frac{(3+h)^2 - 9}{h}$$

$$\lim_{h\to 0} \frac{(3+h)^2-9}{h} = \lim_{h\to 0} \frac{\cancel{3^2} + 2(3)h + h^2 - \cancel{9}}{h}$$

$$= \lim_{h\to 0} \frac{(6+h)h}{h}$$

h's cancel since $(h \neq 0)$.

$$= \lim_{h\to 0} (6+h)$$

$$= 6 + 0$$

$$= \boxed{6}$$

Use the value of the limit above to write down the equation of the tangent line to the graph $y = x^2$ at the point $(3, 9)$.

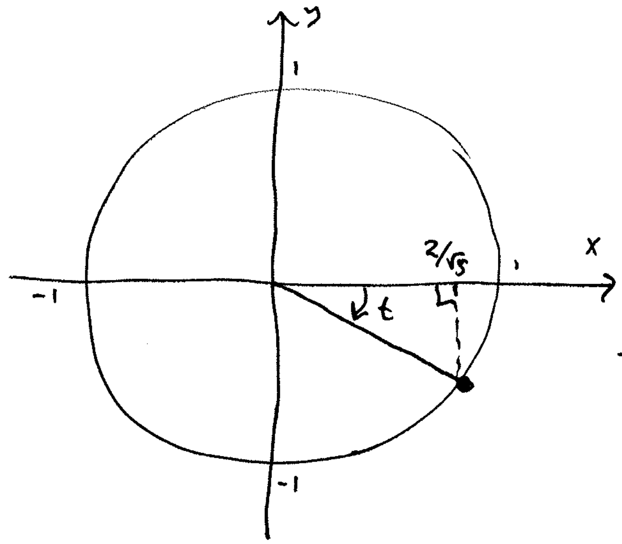
Slope of tangent line $= f'(3) =$ Limit above!

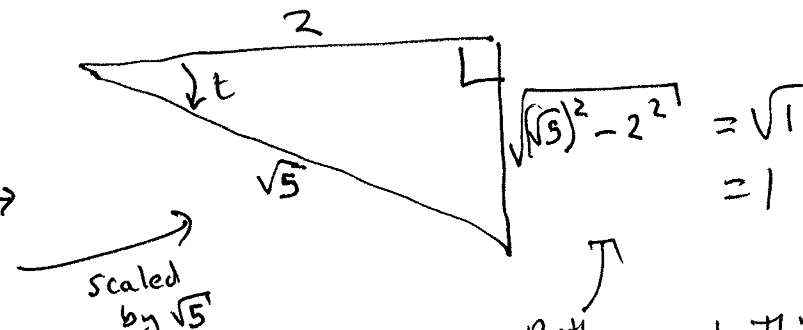
$= 6$.

Point is $(3, 9)$

$\Rightarrow$ Equation is $\boxed{(y-9) = 6(x-3)}$

Q3]... **[13 points]** Suppose that $\cos(t) = 2/\sqrt{5}$ and that $-\pi/2 \le t \le 0$. Draw a picture showing the angle t, and then determine the values of $\sin(t)$ and $\cot(t)$.





Pythagoras' Thm enables us to find 3rd side.
(OR $\cos^2 t + \sin^2 t = 1$)

$$\boxed{\sin(t) = -\frac{1}{\sqrt{5}}}$$

& $\cot(t) = \dfrac{\frac{2}{\sqrt{5}}}{\frac{-1}{\sqrt{5}}} = \dfrac{2}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{-1} = -2$

$$\boxed{\cot(t) = -2}$$

Q4]... **[13 points]** Let f be the piecewise defined function

$$f(x) = \begin{cases} 2 & \text{if } x < 0 \\ x+1 & \text{if } x \ge 0 \end{cases}$$

Is f continuous at 0? Give reasons for your answer.

No...

In order for f to be continuous at 0 we would need $\boxed{\lim_{x \to 0} f(x) = f(0)}$ to hold.

But LHS of this equation does not exist! ⇒ equation doesn't hold true.

⇒ $f(x)$ not continuous at 0.

Why? Because

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (2) = 2$

& $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x+1) = 0+1 = 1$

$2 \neq 1$

$$\Rightarrow \boxed{\lim_{x \to 0} f(x) \text{ DNE}}$$

Q5]... [12 points] Suppose that $f(x)$ is a function whose domain is all real numbers, and suppose that

$$\lim_{x\to 1} \frac{f(x)-8}{x-1} = 2014$$

Use the information above to find the following limit

$$\lim_{x\to 1} f(x)$$

Show the steps of your work.

$$\lim_{x\to 1} (f(x)-8) = \lim_{x\to 1}\left(\frac{(f(x)-8)}{(x-1)}(x-1)\right)$$ ... Legal since $x \neq 1$ ($x-1 \neq 0$).

$$= \lim_{x\to 1}\left(\frac{f(x)-8}{x-1}\right) \cdot \lim_{x\to 1}(x-1)$$

Product Rule for Limits

TOLD THIS; ← Limit Laws ($x-1$ is a polynomial)

$$= (2014) \cdot (1-1)$$

$$= (2014) \cdot 0 = 0$$

So $\lim_{x\to 1} f(x) - \lim_{x\to 1}(8) = 0 \Rightarrow \boxed{\lim_{x\to 1} f(x) = 8}$

Q6]... [12 points] Show that the equation

$$x^5 - x^3 + 3x - 5 = 0$$

has a solution in the interval $(1, 2)$. Give the name of any theorem from class notes that you use.

Let $f(x) = x^5 - x^3 + 3x - 5$

$f(x)$ is continuous at all real numbers (polynomial)

$f(1) = 1 - 1 + 3 - 5 = -2 < 0$

$f(2) = 32 - 8 + 6 - 5 = 25 > 0$

By the Intermediate Value Theorem, there is an input c between 1 & 2 (ie. in $(1,2)$) so that $f(c) = 0$. This is a solution to the equation.