

Friday 10/16/2015

Midterm II

50 minutes

Name: Student ID: **Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	20	
Q2	20	
Q3	20	
Q4	20	
Q5	20	
TOTAL	100	

$$\frac{d}{dk} \ln(\sec k) = \text{tank}$$

Miscellaneous expressions and definitions.

1. Trig Addition, Half Angle.

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

$$\sin^2(x) = (1 - \cos(2x))/2$$

$$\cos^2(x) = (1 + \cos(2x))/2$$

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\sin(2x) = 2\sin(x)\cos(x).$$

2. Hyperbolic.

$$\sinh(x) = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$$

3. Integration by Parts.

$$\int u dv = uv - \int v du$$

4. Inverse Trig.

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

5. Trig Substitutions.

$$\text{For } \sqrt{a^2 - x^2} \text{ use } x = a \sin(\theta)$$

$$\text{For } \sqrt{a^2 + x^2} \text{ use } x = a \tan(\theta)$$

$$\text{For } \sqrt{x^2 - a^2} \text{ use } x = a \sec(\theta)$$

6. Some integrals.

$$\int \tan(x) dx = \ln |\sec(x)| + C$$

$$\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + C$$

Q1]... [20 points] Evaluate the following indefinite integral. Show all your work.

$$\int x \sin(2x) dx$$

Q2]... [20 points] Using a trig substitution (or otherwise) determine the following indefinite integral

$$\int \frac{dx}{\sqrt{x^2 + 4x}}$$

Your answer should be a function of x .

Q3]. . . [20 points] Find constants A, B, C such that

$$\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

Hence evaluate the integral

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$$

Q4]... [20 points] Compute the following limit. Show all the steps of your work.

$$\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$$

Q5]... [20 points] Determine if the following indefinite integral is convergent (in which case, find its value) or divergent.

$$\int_2^{\infty} \frac{dx}{x \ln x}$$