Number Theory Fall 2009
Homework 5
NOT TO BE TURNED IN

Instructions: Here are some additional problems from Chapter 3. While you do not need to turn these in, I recommend doing them for general understanding of the material. Additionally, you may as well expect a problem or two similar to Exercises 3.17–3.19 on the Exam.

3.7 Quadratic Diophantine equations

Exercise 3.16. *Exercise 3.7.1.*

Exercise 3.17. *Show that if $p = x^2 + 4y^2$, then p is of the form $4n$ or $4n + 1$.*

Exercise 3.18. *Using congruences, prove that $k^3 + 2k$ is always divisible by 3.*

Exercise 3.19. *Show that if $p = x^3 + y^3$, then p is not of the form $9n + 4$ or $9n + 5$.*

3.8 *Primitive roots

Proposition 3.16. *If $1/n$ is strictly periodic with period length r, then 10 has order r in $(\mathbb{Z}/n\mathbb{Z})^\times$.*

Exercise 3.20. *Without dividing, determine the period length of $1/11$. (For $p > 5$, we have $gcd(p, 10) = 1$, and one can show $1/p$ will be strictly periodic, so you may use the above proposition.) Check what the decimal expansion is on a calculator.*