Number Theory Fall 2009
Homework 1
Due: Wed. Sep. 2, start of class

Instructions: Read the homework guidelines and policies. Feel free to use a calculator or computer for the computational problems.

Reading assignment

Read the Lecture 1 notes as an overview (which cover some things I didn't have time to say in class), as well as the course guidelines.

Begin reading Chapter 1 of the text. You may also refer to my online notes, but I suggest primarily reading the text as its purpose is to be read (whereas the purpose of my notes are for my lecture).

Written assignment

1.1 Natural Numbers

Definition 1.1. *We say* a divides n *(or* a *is a* divisor *of* n*), and write* $a|n$*, if* $n = ab$*, where* n, a, b *are natural numbers.*

Exercise 1.1. *Using the definition, prove that if* $a|b$ *and* $b|c$*, then* $a|c$ *(transitivity).*

Exercise 1.2. *While there is no known simple way to generate an arbitrary number of primes, certain polynomials are known to produce prime numbers up to a certain point. Let* $p(n) = n^2+n+11$*. Compute* $p(n)$ *for* $0 \leq n \leq 20$*. For which of these values is* $p(n)$ *prime? (Cf. Exercises in 1.1 the text for a similar question.)*

1.2 Induction

Exercise 1.3. *Prove by induction:* $3|2k^3 + k$ *for any natural number* k*.*

1.3 Integers

Exercise 1.4. *1.3.1, 1.3.3 (can use 1.3.2 without doing it), 1.3.4–1.3.6*

1.5 Binary Notation

Exercise 1.5. *Write 19 in binary. Exercises 1.5.1, 1.5.3, 1.5.5.*